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FIRST ORDER LINEAR DIFFERENTIAL EQUATIONS WITH INVOLUTIVE DELAY AND HYPERGEOMETRIC FUNCTIONS

Stefan Czekalski — Henryk Żołądek

Dedicated to the memory of Marek Burnat

Abstract. We present an alternative approach to functions satisfying second order linear ordinary differential equations. It turns out that many of them satisfy a first order ordinary differential equation with an involution. The involution acts on the argument as well as on parameters. Basic examples involve the hypergeometric functions and their descendants.

1. Introduction

Let $t \in \mathbb{C}$ be complex time and $\lambda \in \mathbb{C}^k$ denote parameter(s). Assume that we have an *involution* of $\mathbb{C} \times \mathbb{C}^k$ of the form

$$I\colon (t, \lambda) \mapsto (s, \mu) = (T(t), \Sigma(\lambda)), \tag{1.1}$$

thus $T \circ T = \mathrm{Id}$ and $\Sigma \circ \Sigma = \mathrm{Id}$. By a *linear first order differential equation with an involution* we mean the following equation:

$$\dot{x}_\lambda(t) = a_\lambda(t) x_\mu(s), \tag{1.2}$$

i.e. $\partial x(t;\lambda)/\partial t = a(t;\lambda) \cdot x(T(t);\Sigma(\lambda))$. The function $a(t;\lambda) = a_\lambda(t)$ is called the *directing coefficient.*

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