Topological Methods in Nonlinear Analysis
Volume 51, No. 2, 2018, 583–598
DOI: 10.12775/TMNA.2018.004

A GRADIENT FLOW GENERATED BY A NONLOCAL MODEL OF A NEURAL FIELD IN AN UNBOUNDED DOMAIN

Severino Horacio da Silva — Antônio Luiz Pereira

Abstract. In this paper we consider the nonlocal evolution equation

$$\frac{\partial u(x,t)}{\partial t} + u(x,t) = \int_{\mathbb{R}^N} J(x-y) f(u(y,t))\rho(y)\,dy + h(x).$$

We show that this equation defines a continuous flow in both the space $C_b(\mathbb{R}^N)$ of bounded continuous functions and the space $C_\rho(\mathbb{R}^N)$ of continuous functions u such that $u \cdot \rho$ is bounded, where ρ is a convenient "weight function". We show the existence of an absorbing ball for the flow in $C_b(\mathbb{R}^N)$ and the existence of a global compact attractor for the flow in $C_\rho(\mathbb{R}^N)$, under additional conditions on the nonlinearity. We then exhibit a continuous Lyapunov function which is well defined in the whole phase space and continuous in the $C_\rho(\mathbb{R}^N)$ topology, allowing the characterization of the attractor as the unstable set of the equilibrium point set. We also illustrate our result with a concrete example.

1. Introduction

We consider here the nonlocal evolution equation

$$\frac{\partial u(x,t)}{\partial t} + u(x,t) = \int_{\mathbb{R}^N} J(x-y) f(u(y,t))\rho(y)\,dy + h(x), \tag{1.1}$$

2010 *Mathematics Subject Classification.* Primary: 45J05; Secondary: 37B25.
Key words and phrases. Nonlocal problem; neural field; weighted space; global attractor; Lyapunov functional.
The first author was supported in part by CAPES/CNPq-Brazil Grant 552464/2012-2.
The second author was supported in part by FAPESP-Brazil Grant 2016/02150-8.