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A VERSION OF KRASNOSEL'SKII'S COMPRESSION–EXPANSION FIXED POINT THEOREM IN CONES FOR DISCONTINUOUS OPERATORS WITH APPLICATIONS

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Abstract. We introduce a new fixed point theorem of Krasnosel'skiĭ type for discontinuous operators. As an application we use it to study the existence of positive solutions of a second-order differential problem with separated boundary conditions and discontinuous nonlinearities.

1. Introduction

A classical problem [11], [12], [14] is that of the existence of positive solutions for the differential equation

$$u''(t) + g(t)f(u(t)) = 0, \quad 0 < t < 1, \tag{1.1}$$

along with suitable boundary conditions (BCs). This problem arises in the study of radial solutions in $\mathbb{R}^n$, $n \geq 2$, for the partial differential equation (PDE)

$$\Delta v + h(\|x\|)f(v) = 0, \quad x \in \mathbb{R}^n, \ \|x\| \in [R_1, R_2],$$

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