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ON A SINGULAR SEMILINEAR ELLIPTIC PROBLEM: MULTIPLE SOLUTIONS VIA CRITICAL POINT THEORY

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Abstract. We study existence and multiplicity of solutions of a semilinear elliptic problem involving a singular term. Combining various techniques from critical point theory, under different sets of assumptions, we prove the existence of k solutions ($k \in \mathbb{N}$) or infinitely many weak solutions.

1. Introduction and statement of results

In the present paper we deal with the following semilinear elliptic problem involving a singular term:

$$(\mathcal{P}) \qquad \begin{cases} -\Delta u = f(u) + u^{-\gamma} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ ($N > 2$) with smooth boundary $\partial\Omega$, $f\colon [0, +\infty[\to \mathbb{R}$ is a continuous function and $0 < \gamma < 1$. The existence of multiple weak solutions is established under various assumptions on the nonlinearity f by combining different techniques from critical point theory. We remark that the energy functional associated to $(\mathcal{P})$ is not in general of class C^1 and this causes an obstacle to the application of such a theory.

The study of singular elliptic problems started with the pioneering work of Fulks and Maybee ([8]) as a mathematical model for describing the heat

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