ON A SINGULAR SEMILINEAR ELLIPTIC PROBLEM: MULTIPLE SOLUTIONS VIA CRITICAL POINT THEORY

FRANCESCO FARACI — GEORGE SMYRLOS

Abstract. We study existence and multiplicity of solutions of a semilinear elliptic problem involving a singular term. Combining various techniques from critical point theory, under different sets of assumptions, we prove the existence of $k$ solutions ($k \in \mathbb{N}$) or infinitely many weak solutions.

1. Introduction and statement of results

In the present paper we deal with the following semilinear elliptic problem involving a singular term:

\[
(P) \quad \begin{cases} 
-\Delta u = f(u) + u^{-\gamma} & \text{in } \Omega, \\
 u > 0 & \text{in } \Omega, \\
 u = 0 & \text{on } \partial \Omega,
\end{cases}
\]

where $\Omega$ is a bounded domain in $\mathbb{R}^N$ ($N > 2$) with smooth boundary $\partial \Omega$, $f: [0, +\infty[ \to \mathbb{R}$ is a continuous function and $0 < \gamma < 1$. The existence of multiple weak solutions is established under various assumptions on the nonlinearity $f$ by combining different techniques from critical point theory. We remark that the energy functional associated to $(P)$ is not in general of class $C^1$ and this causes an obstacle to the application of such a theory.

The study of singular elliptic problems started with the pioneering work of Fulks and Maybee ([8]) as a mathematical model for describing the heat

2010 Mathematics Subject Classification. 35J65, 35J20.

Key words and phrases. Singular elliptic problem; multiple solutions; critical point theory.