EXISTENCE OF A WEAK SOLUTION
FOR THE FRACTIONAL p-LAPLACIAN EQUATIONS
WITH DISCONTINUOUS NONLINEARITIES
VIA THE BERKOVITS–TIENARI DEGREE THEORY

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ABSTRACT. We are concerned with the following nonlinear elliptic equations of the fractional p-Laplace type:

\[
\begin{aligned}
&\bigl( (-\Delta)^{\alpha}u \bigr) \in \lambda f(x, u(x)), \quad \bar{f}(x, u(x)) \bigr) \\
&\quad \text{in } \Omega, \\
&\quad u = 0 \\
&\quad \text{on } \mathbb{R}^N \setminus \Omega,
\end{aligned}
\]

where \((-\Delta)^{\alpha}\) is the fractional p-Laplacian operator, \(\lambda\) is a parameter, \(0 < \alpha < 1 < p < +\infty\), \(sp < N\), and the measurable functions \(f, \bar{f}\) are induced by a possibly discontinuous at the second variable function \(f: \Omega \times \mathbb{R} \to \mathbb{R}\). By using the Berovits–Tienari degree theory for upper semicontinuous set-valued operators of type \((S_+)\) in reflexive Banach spaces, we show that our problem with the discontinuous nonlinearity \(f\) possesses at least one nontrivial weak solution. In addition, we show the existence of two nontrivial weak solutions in which one has negative energy and another has positive energy.

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