Topological **M**ethods in **N**onlinear **A**nalysis
Volume 51, No. 2, 2018, 371–388
DOI: 10.12775/TMNA.2017.064

EXISTENCE OF A WEAK SOLUTION FOR THE FRACTIONAL p-LAPLACIAN EQUATIONS WITH DISCONTINUOUS NONLINEARITIES VIA THE BERKOVITS–TIENARI DEGREE THEORY

Yun-Ho Kim

Abstract. We are concerned with the following nonlinear elliptic equations of the fractional p-Laplace type:

$$\begin{cases} (-\Delta)_p^s u \in \lambda[\underline{f}(x,u(x)), \overline{f}(x,u(x))] & \text{in } \Omega, \\ u = 0 & \text{on } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $(-\Delta)_p^s$ is the fractional p-Laplacian operator, λ is a parameter, $0 < s < 1 < p < +\infty$, $sp < N$, and the measurable functions $\underline{f}$, $\overline{f}$ are induced by a possibly discontinuous at the second variable function $f\colon \Omega \times \mathbb{R} \to \mathbb{R}$. By using the Berkovits–Tienari degree theory for upper semicontinuous set-valued operators of type (S_+) in reflexive Banach spaces, we show that our problem with the discontinuous nonlinearity f possesses at least one nontrivial weak solution. In addition, we show the existence of two nontrivial weak solutions in which one has negative energy and another has positive energy.

2010 *Mathematics Subject Classification.* Primary: 35R11, 35J60; Secondary: 47H11.

Key words and phrases. Fractional p-Laplacian; weak solution; critical point; degree theory.

The author is grateful to the referees for their valuable comments and suggestions for improvement of the paper. This research was supported by a 2015 Research Grant from Sangmyung University.