EXISTENCE OF THREE NONTRIVIAL SOLUTIONS FOR A CLASS OF FOURTH-ORDER ELLIPTIC EQUATIONS

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ABSTRACT. The existence of three nontrivial solutions is established for a class of fourth-order elliptic equations. Our technical approach is based on Linking Theorem and $(\nabla)$-Theorem.

1. Introduction and main results

We consider the fourth order elliptic equation

\[\begin{aligned}
\Delta^2 u + c\Delta u &= \mu u + f(x, u) & \text{in } \Omega, \\
u &= \Delta u = 0 & \text{on } \partial\Omega,
\end{aligned}\]

where $\Omega \subset \mathbb{R}^N$ ($N > 4$) is a bounded smooth domain, $c \in \mathbb{R}$ and $f: \Omega \times \mathbb{R} \to \mathbb{R}$. $\Delta$ is the Laplace operator and $\Delta^2$ is the biharmonic operator.

Let $0 < \lambda_1 < \ldots < \lambda_k < \ldots$ be the distinct eigenvalues of $-\Delta$ in $H^1_0(\Omega)$. The eigenvalue problem

\[\begin{aligned}
\Delta^2 u + c\Delta u &= \mu u & \text{in } \Omega, \\
u &= \Delta u = 0 & \text{on } \partial\Omega,
\end{aligned}\]

2010 Mathematics Subject Classification. Primary: 35J35, 35J65; Secondary: 35J61, 35B38.

Key words and phrases. Fourth-order elliptic equations; linking theorem; $(\nabla)$-theorem; critical points.

Supported by the Natural Science Foundation of Chongqing (No. cstc2017jcyJA0044), the National Natural Science Foundation of China (No. 11471267), the Fundamental Research Funds for the Central Universities (No. XDJK2014B041, No. SWU115033).