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POISSON STRUCTURES ON CLOSED MANIFOLDS

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Abstract. We prove an h-principle for Poisson structures on closed manifolds. Equivalently, we prove an h-principle for symplectic foliations (singular) on closed manifolds. On open manifolds however the singularities could be avoided and it is a known result by Fernandes and Frejlich [7].

1. Introduction

In this paper we prove an h-principle for Poisson structures on closed manifolds. Similar results on open manifolds have been proved by Fernandes and Frejlich in [7]. We recall their result below.

Let M^{2n+q} be a C^∞-manifold equipped with a co-dimension-q foliation $\mathcal{F}_0$ and a 2-form ω_0 such that $(\omega_0^n)_{|T\mathcal{F}_0} \neq 0$. Denote by $\mathrm{Fol}_q(M)$ the space of co-dimension-q foliations on M identified with a subspace of $\Gamma(\mathrm{Gr}_{2n}(M))$, where $\mathrm{Gr}_{2n}(M) \xrightarrow{\mathrm{pr}} M$ is the Grassmann bundle, i.e. $\mathrm{pr}^{-1}(x) = \mathrm{Gr}_{2n}(T_xM)$ and $\Gamma(\mathrm{Gr}_{2n}(M))$ is the space of sections of $\mathrm{Gr}_{2n}(M) \xrightarrow{\mathrm{pr}} M$ with compact open topology. Define

$$\Delta_q(M) \subset \mathrm{Fol}_q(M) \times \Omega^2(M), \qquad \Delta_q(M) := \{(\mathcal{F}, \omega) : \omega^n_{|T\mathcal{F}} \neq 0\}.$$

Obviously $(\mathcal{F}_0, \omega_0) \in \Delta_q(M)$. In this setting Fernandes and Frejlich proved the following

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