Topological **M**ethods in **N**onlinear **A**nalysis
Volume 51, No. 1, 2018, 215–242
DOI: 10.12775/TMNA.2017.046

GROUND STATE SOLUTIONS FOR A CLASS OF SEMILINEAR ELLIPTIC SYSTEMS WITH SUM OF PERIODIC AND VANISHING POTENTIALS

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Abstract. In this paper, we consider the following semilinear elliptic systems:

$$\begin{cases} -\Delta u + V(x)u = F_u(x,u,v) - \Gamma(x)|u|^{q-2}u & \text{in } \mathbb{R}^N, \\ -\Delta v + V(x)v = F_v(x,u,v) - \Gamma(x)|v|^{q-2}v & \text{in } \mathbb{R}^N, \end{cases}$$

where $q \in [2, 2^*)$, $V = V_{\rm per} + V_{\rm loc} \in L^\infty(\mathbb{R}^N)$ is the sum of a periodic potential $V_{\rm per}$ and a localized potential $V_{\rm loc}$ and $\Gamma \in L^\infty(\mathbb{R}^N)$ is periodic and $\Gamma(x) \geq 0$ for almost every $x \in \mathbb{R}^N$. Under some appropriate assumptions on F, we investigate the existence and nonexistence of ground state solutions for the above system. Recent results from the literature are improved and extended.

1. Introduction

In this paper, we consider the existence and nonexistence of ground state solutions to the following semilinear elliptic systems:

$$\begin{cases} -\Delta u + V(x)u = F_u(x,u,v) - \Gamma(x)|u|^{q-2}u & \text{in } \mathbb{R}^N, \\ -\Delta v + V(x)v = F_v(x,u,v) - \Gamma(x)|v|^{q-2}v & \text{in } \mathbb{R}^N, \end{cases} \tag{1.1}$$

2010 *Mathematics Subject Classification.* 35B38, 35J20.

Key words and phrases. Semilinear elliptic systems; ground state; periodic potential; localized potential; variational methods.

This work is partially supported by the National Natural Science Foundation of China 11671403, by the Fundamental Research Funds for the Central Universities of Central South University 2017zzts058 and by the Mathematics and Interdisciplinary Sciences Project of CSU.