GROUND STATE SOLUTIONS
FOR A CLASS OF SEMILINEAR ELLIPTIC SYSTEMS
WITH SUM OF PERIODIC AND VANISHING POTENTIALS

GUOFENG CHE — HAIHO CHEN

ABSTRACT. In this paper, we consider the following semilinear elliptic systems:

\[
\begin{align*}
-\Delta u + V(x)u &= F_u(x,u,v) - \Gamma(x)|u|^{q-2}u \quad \text{in } \mathbb{R}^N, \\
-\Delta v + V(x)v &= F_v(x,u,v) - \Gamma(x)|v|^{q-2}v \quad \text{in } \mathbb{R}^N,
\end{align*}
\]

where \( q \in [2,2^*) \), \( V = V_{\text{per}} + V_{\text{loc}} \in L^\infty(\mathbb{R}^N) \) is the sum of a periodic potential \( V_{\text{per}} \) and a localized potential \( V_{\text{loc}} \), and \( \Gamma \in L^\infty(\mathbb{R}^N) \) is periodic and \( \Gamma(x) \geq 0 \) for almost every \( x \in \mathbb{R}^N \). Under some appropriate assumptions on \( F \), we investigate the existence and nonexistence of ground state solutions for the above system. Recent results from the literature are improved and extended.

1. Introduction

In this paper, we consider the existence and nonexistence of ground state solutions to the following semilinear elliptic systems:

\[
\begin{align*}
-\Delta u + V(x)u &= F_u(x,u,v) - \Gamma(x)|u|^{q-2}u \quad \text{in } \mathbb{R}^N, \\
-\Delta v + V(x)v &= F_v(x,u,v) - \Gamma(x)|v|^{q-2}v \quad \text{in } \mathbb{R}^N,
\end{align*}
\]

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