Topological **M**ethods in **N**onlinear **A**nalysis
Volume 51, No. 1, 2018, 135–150
DOI: 10.12775/TMNA.2017.041

ON THE WELL-POSEDNESS OF DIFFERENTIAL MIXED QUASI-VARIATIONAL-INEQUALITIES

Zhenhai Liu — Dumitru Motreanu — Shengda Zeng

Abstract. We discuss the well-posedness and the well-posedness in the generalized sense of differential mixed quasi-variational inequalities ((DMQVIs), for short) in Hilbert spaces. This gives us an outlook to the convergence analysis of approximating sequences of solutions for (DMQVIs). Using these concepts we point out the relation between metric characterizations and well-posedness of (DMQVIs). We also prove that the solution set of (DMQVIs) is compact, if problem (DMQVIs) is well-posed in the generalized sense.

1. Introduction

Let X be a Hilbert space whose norm and scalar product are $\|\cdot\|_X$ and $\langle\cdot,\cdot\rangle_X$, respectively. The norm convergence is denoted by $\to$ and the weak convergence by $\rightharpoonup$. Let $0 < T < +\infty$ and $I := [0,T]$. Recall that the Hilbert space $L^2(I;X)$ is endowed with the scalar product defined by

$$\langle u_1, u_2\rangle_{L^2(I;X)} := \int_0^T \langle u_1(t), u_2(t)\rangle_X \, dt, \quad \text{for all } u_1, u_2 \in L^2(I;X).$$

2010 *Mathematics Subject Classification.* 47H04, 49J53, 90C31.

Key words and phrases. Differential mixed quasi-variational inequalities; well-posedness; approximating sequence; relaxed α-monotonicity.

Project supported by NNSF of China Grant No. 11671101, the National Science Center of Poland under Maestro Advanced Project No. UMO-2012/06/A/ST1/00262, Special Funds of Guangxi Distinguished Experts Construction Engineering.