NONHOMOGENEOUS DIRICHLET PROBLEMS WITHOUT THE AMBROSETTI–RABINOWITZ CONDITION

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ABSTRACT. We consider the existence of solutions of the following $p(x)$-Laplacian Dirichlet problem without the Ambrosetti–Rabinowitz condition:

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u) = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

We give a new growth condition and we point out its importance for checking the Cerami compactness condition. We prove the existence of solutions of the above problem via the critical point theory, and also provide some multiplicity properties. The present paper extend previous results of Q. Zhang and C. Zhao (Existence of strong solutions of a $p(x)$-Laplacian Dirichlet problem without the Ambrosetti–Rabinowitz condition, *Computers and Mathematics with Applications*, 2015) and we establish the existence of solutions under weaker hypotheses on the nonlinear term.