LINEAR QUADRATIC GAME OF EXPLOITATION
OF COMMON RENEWABLE RESOURCES
WITH INHERENT CONSTRAINTS

RAJANI SINGH — AGNIESZKA WISZNIEWSKA-MATYSZKIEL

ABSTRACT. In this paper, we analyse a linear quadratic multistage game
of extraction of a common renewable resource — a fishery — by many
players with inherent state dependent constraints for exploitation and an
infinite time horizon. To the best of our knowledge, such games have never
been studied. We analyse the social optimum and Nash equilibrium for the
feedback information structure and compare the results obtained in both
cases. For the Nash equilibria, we obtain a value function that is contrary
to intuitions from standard linear quadratic games. In our game, we face
a situation in which the social optimum results in sustainability, while the
Nash equilibrium leads to the depletion of the fishery in a finite time for
realistic levels of the initial biomass of fish. Therefore, we also study an
introduction of a tax in order to enforce socially optimal behaviour of the
players. Besides, this game constitutes a counterexample to simplifications
of techniques often used in computation of Nash equilibria and/or optimal
control problems.

1. Introduction

Exploitation of common or interdependent renewable resources without an
exclusive owner is a very important problem of contemporaneity.