

PROPERTIES OF UNIQUE POSITIVE SOLUTIONS FOR A CLASS OF NONLOCAL SEMILINEAR ELLIPTIC EQUATIONS

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ABSTRACT. We study a class of nonlocal elliptic equations

$$-M\left(\int_{\Omega}|u|^{\gamma}dx\right)\Delta u = \lambda f(x, u)$$

with the Dirichlet boundary conditions in bounded domain. Under suitable assumptions on M and the nonlinear term f , the existence and new properties of a unique positive solutions are obtained via a monotone operator method and a mixed monotone operator method.

1. Introduction

In this article, we consider the following nonlocal elliptic problem:

$$(1.1) \quad \begin{cases} -M\left(\int_{\Omega}|u|^{\gamma}dx\right)\Delta u = \lambda f(x, u) & \text{for } x \in \Omega, \\ u(x) = 0 & \text{for } x \in \partial\Omega, \end{cases}$$

where $\Omega \subseteq \mathbb{R}^N$ ($N \geq 1$) is a smooth and bounded domain, $\gamma \in (0, +\infty)$, $\lambda > 0$ is a parameter. $M: [0, +\infty) \rightarrow (0, +\infty)$ and $f: \bar{\Omega} \times (0, +\infty) \rightarrow [0, +\infty)$ are given functions whose properties will be listed later.

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