MAYER–VIETORIS PROPERTY
OF THE FIXED POINT INDEX

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ABSTRACT. We study a Mayer–Vietoris kind formula for the fixed point index
of maps of ENR triplets \( f : (X; X_1, X_2) \to (X; X_1, X_2) \) having compact
fixed point set. We prove it under some suitable conditions. For instance
when \( (X; X_1, X_2) = (E^n; E^n_0, E^n_0) \).
We use these results to generalize the Poincaré–Bendixson index formula for
vector fields to continuous maps having a sectorial decomposition, to study
the fixed point index \( i(f, 0) \) of orientation preserving homeomorphisms of
\( E^2_+ \) and \( (B^3; E^3_1, E^3_0) \) and the fixed point index in the invariant subspace.

1. Introduction

In this paper we will deal with triplets \( (X; X_1, X_2) \) where \( X \) is an ENR
and \( X_1, X_2 \) are ENR's closed in \( X \) such that \( X_1 \cap X_2 \) is also an ENR and
\( X = X_1 \cup X_2 \). We will call such triplets \( (X; X_1, X_2) \) ENR triplets and we will
denote \( X_0 := X_1 \cap X_2 \). A continuous map \( f : (X; X_1, X_2) \to (X; X_1, X_2) \) of
a triplet is a continuous map \( f : X \to X \) satisfying that
\[
f(X_i) \subset X_i, \quad i = 1, 2.
\]
Notice that \( f(X_0) \subset X_0 \).

We denote
\[
f_i := f|_{X_i} : X_i \to X_i, \quad i = 0, 1, 2.
\]

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proper pair; isolated invariant set.