MAYER–VIETORIS PROPERTY
OF THE FIXED POINT INDEX

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ABSTRACT. We study a Mayer–Vietoris kind formula for the fixed point index of maps of ENR triplets $f\colon (X; X_1, X_2) \to (X; X_1, X_2)$ having compact fixed point set. We prove it under some suitable conditions. For instance when $(X; X_1, X_2) = (E^n; E^n_+, E^n_-)$.

We use these results to generalize the Poincaré–Bendixson index formula for vector fields to continuous maps having a *sectorial decomposition*, to study the fixed point index $i(f, 0)$ of orientation preserving homeomorphisms of E^2_+ and $(E^3; E^3_+, E^3_-)$ and the fixed point index in the invariant subspace.

1. Introduction

In this paper we will deal with triplets $(X; X_1, X_2)$ where X is an ENR and X_1, X_2 are ENR's closed in X such that $X_1 \cap X_2$ is also an ENR and $X = X_1 \cup X_2$. We will call such triplets $(X; X_1, X_2)$ *ENR triplets* and we will denote $X_0 := X_1 \cap X_2$. A continuous map $f\colon (X; X_1, X_2) \to (X; X_1, X_2)$ of a triplet is a continuous map $f\colon X \to X$ satisfying that

$$f(X_i) \subset X_i, \quad i = 1, 2.$$

Notice that $f(X_0) \subset X_0$.

We denote

$$f_i := f|_{X_i}\colon X_i \to X_i, \quad i = 0, 1, 2.$$