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CONCENTRATION OF GROUND STATE SOLUTIONS FOR FRACTIONAL HAMILTONIAN SYSTEMS

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Abstract. We are concerned with the existence of ground states solutions to the following fractional Hamiltonian systems:

$$(\mathrm{FHS})_\lambda \qquad \begin{cases} -{}_tD_\infty^\alpha({}_{-\infty}D_t^\alpha u(t)) - \lambda L(t)u(t) + \nabla W(t,u(t)) = 0, \\ u \in H^\alpha(\mathbb{R},\mathbb{R}^n), \end{cases}$$

where $\alpha \in (1/2,1)$, $t \in \mathbb{R}$, $u \in \mathbb{R}^n$, $\lambda > 0$ is a parameter, $L \in C(\mathbb{R},\mathbb{R}^{n^2})$ is a symmetric matrix for all $t \in \mathbb{R}$, $W \in C^1(\mathbb{R}\times\mathbb{R}^n,\mathbb{R})$ and $\nabla W(t,u)$ is the gradient of $W(t,u)$ at u. Assuming that $L(t)$ is a positive semi-definite symmetric matrix for all $t \in \mathbb{R}$, that is, $L(t) \equiv 0$ is allowed to occur in some finite interval T of $\mathbb{R}$, $W(t,u)$ satisfies the Ambrosetti–Rabinowitz condition and some other reasonable hypotheses, we show that $(\mathrm{FHS})_\lambda$ has a ground sate solution which vanishes on $\mathbb{R}\setminus T$ as $\lambda \to \infty$, and converges to $u \in H^\alpha(\mathbb{R},\mathbb{R}^n)$, where $u \in E_0^\alpha$ is a ground state solution of the Dirichlet BVP for fractional systems on the finite interval T. Recent results are generalized and significantly improved.

1. Introduction

Fractional differential equations both ordinary and partial ones are applied in mathematical modeling of processes in physics, mechanics, control theory, biochemistry, bioengineering and economics. Therefore, the theory of fractional

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