SINGULARLY PERTURBED $N$-LAPLACIAN PROBLEMS WITH A NONLINEARITY IN THE CRITICAL GROWTH RANGE

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ABSTRACT. We consider the following singularly perturbed problem:

$$-\varepsilon^N \Delta_N u + V(x)|u|^{N-2}u = f(u), \quad u(x) > 0 \quad \text{in } \mathbb{R}^N,$$

where $N \geq 2$ and $\Delta_N u$ is the $N$-Laplacian operator. In this paper, we construct a solution $u_\varepsilon$ which concentrates around any given isolated positive local minimum component of $V$, as $\varepsilon \to 0$, in the Trudinger-Moser type of subcritical or critical case. In the subcritical case, we only impose on $f$ the Berestycki and Lions conditions. In the critical case, a global condition on the nonlinearity $f$ is imposed. However, any monotonicity of $f(t)/t^{N-1}$ or Ambrosetti-Rabinowitz type conditions are not required.

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553