EXISTENCE OF MULTIPLE SOLUTIONS
FOR A QUASILINEAR ELLIPTIC PROBLEM

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ABSTRACT. In this paper we prove the existence of multiple solutions for
a quasilinear elliptic boundary value problem, when the
$p$-derivative at zero and the $p$-derivative at infinity of
the nonlinearity are greater than
the first eigenvalue of the $p$-Laplace operator.
Our proof uses bifurcation
from infinity and bifurcation from zero to prove the existence of unbounded
branches of positive solutions (resp. of negative solutions).
We show the
existence of multiple solutions and we provide qualitative properties of these
solutions.

1. Introduction

In this paper we study the existence of multiple solutions for the quasilinear
elliptic boundary value problem

\begin{equation}
\begin{cases}
\Delta_p u + f(u) = 0 & \text{in } \Omega, \\
u = 0 & \text{on } \partial \Omega,
\end{cases}
\end{equation}

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a bounded and smooth domain, $1 < p < 2$, and
$f: \mathbb{R} \to \mathbb{R}$ is a nonlinear function such that $f(0) = 0$ and

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