

EXISTENCE OF MULTIPLE SOLUTIONS FOR A QUASILINEAR ELLIPTIC PROBLEM

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ABSTRACT. In this paper we prove the existence of multiple solutions for a quasilinear elliptic boundary value problem, when the p -derivative at zero and the p -derivative at infinity of the nonlinearity are greater than the first eigenvalue of the p -Laplace operator. Our proof uses bifurcation from infinity and bifurcation from zero to prove the existence of unbounded branches of positive solutions (resp. of negative solutions). We show the existence of multiple solutions and we provide qualitative properties of these solutions.

1. Introduction

In this paper we study the existence of multiple solutions for the quasilinear elliptic boundary value problem

$$(1.1) \quad \begin{cases} \Delta_p u + f(u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a bounded and smooth domain, $1 < p < 2$, and $f: \mathbb{R} \rightarrow \mathbb{R}$ is a nonlinear function such that $f(0) = 0$ and

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