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EXISTENCE THEORY FOR QUASILINEAR ELLIPTIC EQUATIONS VIA A REGULARIZATION APPROACH

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Abstract. In this paper, we further develop a regularization approach initiated in our earlier work for the study of solution structure of quasilinear elliptic equations containing several special cases of mathematical models.

1. Introduction

We consider the following quasilinear elliptic equation:

$$\begin{cases} \displaystyle\sum_{i,j=1}^{N} D_j(a_{ij}(x,u)D_iu) \\ \qquad \displaystyle -\frac{1}{2}\sum_{i,j=1}^{N} D_s a_{ij}(x,u)D_iuD_ju + f(x,u) = 0 \quad \text{in } \Omega, \\ u = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain,

$$D_i = \frac{\partial}{\partial x_i}, \qquad D_s a_{ij}(x,s) = \frac{\partial}{\partial s}\, a_{ij}(x,s), \qquad a_{ij} = a_{ji}.$$

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