Topological **M**ethods in **N**onlinear **A**nalysis
Volume 50, No. 2, 2017, 357–468
DOI: 10.12775/TMNA.2016.072

RIGOROUS NUMERICS FOR FAST-SLOW SYSTEMS WITH ONE-DIMENSIONAL SLOW VARIABLE: TOPOLOGICAL SHADOWING APPROACH

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Abstract. We provide a rigorous numerical computational method to validate periodic, homoclinic and heteroclinic orbits as the continuation of singular limit orbits for the fast-slow system

$$\begin{cases} x' = f(x, y, \varepsilon), \\ y' = \varepsilon g(x, y, \varepsilon) \end{cases}$$

with one-dimensional slow variable y. Our validation procedure is based on topological tools called isolating blocks, cone conditions and covering relations. Such tools provide us with existence theorems of global orbits which shadow singular orbits in terms of a new concept, the covering-exchange. Additional techniques called slow shadowing and m-cones are also developed. These techniques give us not only generalized topological verification theorems, but also easy implementations for validating trajectories near slow manifolds in a wide range, via rigorous numerics. Our procedure is available to validate global orbits not only for sufficiently small $\varepsilon > 0$ but all ε in a given half-open interval $(0, \varepsilon_0]$. Several sample verification examples are shown as a demonstration of applicability.

2010 *Mathematics Subject Classification.* 34E15, 37B25, 37C29, 37C50, 37D10, 65L11.

Key words and phrases. Singular perturbations; periodic orbits; connecting orbits; rigorous numerics; covering-exchange.