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NONLINEAR DELAY REACTION-DIFFUSION SYSTEMS WITH NONLOCAL INITIAL CONDITIONS HAVING AFFINE GROWTH

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(*Submitted by W. Kryszewski*)

Abstract. We consider a class of abstract evolution reaction-diffusion systems with delay and nonlocal initial data of the form

$$\begin{cases} u'(t) \in Au(t) + F(t, u_t, v_t) & \text{for } t \in \mathbb{R}_+,\\ v'(t) \in Bv(t) + G(t, u_t, v_t) & \text{for } t \in \mathbb{R}_+,\\ u(t) = p(u,v)(t) & \text{for } t \in [-\tau_1, 0],\\ v(t) = q(u,v)(t) & \text{for } t \in [-\tau_2, 0], \end{cases}$$

where $\tau_i \geq 0$, $i = 1, 2$, A and B are two m-dissipative operators acting in two Banach spaces, the perturbations F and G are continuous, while the history functions p and q are nonexpansive functions with affine growth. We prove an existence result of C^0-solutions for the above problem and we give an example to illustrate the effectiveness of our abstract theory.

1. Introduction

Let X, Y be Banach spaces, $\tau_1, \tau_2 \geq 0$, and let $A\colon D(A) \subseteq X \rightsquigarrow X$ and $B\colon D(B) \subseteq Y \rightsquigarrow Y$ be m-dissipative operators. Our paper is devoted to provide

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an existence result for C^0-solutions to the next reaction-diffusion system with delay and nonlocal initial conditions:

$$
\begin{cases}
u'(t) \in Au(t) + F(t, u_t, v_t) & \text{for } t \in \mathbb{R}_+,\\
v'(t) \in Bv(t) + G(t, u_t, v_t) & \text{for } t \in \mathbb{R}_+,\\
u(t) = p(u, v)(t) & \text{for } t \in [-\tau_1, 0],\\
v(t) = q(u, v)(t) & \text{for } t \in [-\tau_2, 0],
\end{cases} \tag{1.1}
$$

where the perturbations $F\colon \mathbb{R}_+ \times C([-\tau_1, 0]; \overline{D(A)}) \times C([-\tau_2, 0]; \overline{D(B)}) \to X$ and $G\colon \mathbb{R}_+ \times C([-\tau_1, 0]; \overline{D(A)}) \times C([-\tau_2, 0]; \overline{D(B)}) \to Y$ are continuous and the initial data $p\colon C_{\rm b}([-\tau_1, +\infty); \overline{D(A)}) \times C_{\rm b}([-\tau_2, +\infty); \overline{D(B)}) \to C([-\tau_1, 0]; X)$ and $q\colon C_{\rm b}([-\tau_1, +\infty); \overline{D(A)}) \times C_{\rm b}([-\tau_2, +\infty); \overline{D(B)}) \to C([-\tau_2, 0]; Y)$ are nonexpansive functions with affine growth.

Partial differential equations with nonlocal initial conditions arise in many areas of applied mathematics and represent mathematical models of various phenomena. See Deng [18] and McKibben [25]. The study for nonlocal Cauchy problems without delay was initiated by Byszewski [15] (in the semilinear case), and subsequently it has been developed by many authors. We mention here some significant contributions to the field: Aizicovici and Lee [1], Aizicovici and McKibben [2], García-Falset [21], García-Falset and Reich [22], Cardinali, Precup and Rubbioni [16] in the single-valued case, Aizicovici and Staicu [3], Paicu and Vrabie [32], Zhu and Li [43] in the multi-valued case. Nica [31] proved the existence of the solutions for nonlinear first order differential systems with nonlocal conditions. These results were extended by Bolojan-Nica, Infante and Precup [7] to differential systems with nonlinear and nonlocal boundary conditions. For delay evolution equations with local initial conditions see Mitidieri and Vrabie [26], [27], Necula and Popescu [28], and the references therein. As far as nonlocal initial conditions are concerned, we mention the papers Burlică and Roşu [11], Burlică, Roşu and Vrabie [13], Necula, Popescu and Vrabie [29], Vrabie [37]–[41], Wang and Zhu [42]. For parabolic systems with nonlinear, nonlocal initial conditions we mention the paper of Infante and Maciejewski [24]. Concerning the reaction-diffusion systems without delay see: Burlică [8], Burlică and Roşu [9], [10], Díaz and Vrabie [19], Necula and Vrabie [30], Roşu [33], [34]. Existence results for reaction-diffusion systems with delay and nonlocal initial conditions were obtained in Burlică, Roşu and Vrabie [14] for the single-valued case and by Burlică and Roşu [12] for the multi-valued case. The present work complements Burlică, Roşu and Vrabie [14] by allowing the nonlocal initial constraint function p to have affine instead of linear growth with respect to the first argument and q to obey the same property with respect to its second variable. Moreover, we allow the unknown functions to have different delays, τ_1 and τ_2. Our general assumptions include reaction-diffusion systems in which one or both