NONLINEAR DELAY REACTION-DIFFUSION SYSTEMS
WITH NONLOCAL INITIAL CONDITIONS
HAVING AFFINE GROWTH

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(Submitted by W. Kryszewski)

ABSTRACT. We consider a class of abstract evolution reaction-diffusion systems with delay and nonlocal initial data of the form

\[
\begin{cases}
  u'(t) \in Au(t) + F(t, u_t, u_{\tau_1}) & \text{for } t \in \mathbb{R}_+,
  \\
  v'(t) \in Bv(t) + G(t, u_t, u_{\tau_2}) & \text{for } t \in \mathbb{R}_+,
  \\
  u(t) = p(u, v)(t) & \text{for } t \in [-\tau_1, 0],
  \\
  v(t) = q(u, v)(t) & \text{for } t \in [-\tau_2, 0],
\end{cases}
\]

where \( \tau_i > 0, i = 1, 2 \), \( A \) and \( B \) are two m-dissipative operators acting in two Banach spaces, the perturbations \( F \) and \( G \) are continuous, while the history functions \( p \) and \( q \) are nonexpansive functions with affine growth. We prove an existence result of \( C^0 \)-solutions for the above problem and we give an example to illustrate the effectiveness of our abstract theory.

1. Introduction

Let \( X, Y \) be Banach spaces, \( \tau_1, \tau_2 \geq 0 \), and let \( A: D(A) \subseteq X \rightharpoonup X \) and \( B: D(B) \subseteq Y \rightharpoonup Y \) be m-dissipative operators. Our paper is devoted to provide

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an existence result for $C^0$-solutions to the next reaction-diffusion system with delay and nonlocal initial conditions:

\[
\begin{aligned}
    u'(t) &\in Au(t) + F(t, u_t, u_t) \quad \text{for } t \in \mathbb{R}_+,
    \\
v'(t) &\in Bu(t) + G(t, u_t, u_t) \quad \text{for } t \in \mathbb{R}_+,
    \\
u(t) & = p(u, v)(t) \quad \text{for } t \in [-\tau_1, 0],
    \\
v(t) & = q(u, v)(t) \quad \text{for } t \in [-\tau_2, 0],
\end{aligned}
\]  

(1.1)

where the perturbations $F: \mathbb{R}_+ \times C([-\tau_1, 0]; \overline{D(A)}) \times C([-\tau_2, 0]; \overline{D(B)}) \to X$ and $G: \mathbb{R}_+ \times C([-\tau_1, 0]; \overline{D(A)}) \times C([-\tau_2, 0]; \overline{D(B)}) \to Y$ are continuous and the initial data $p: C_b([0, +\infty); \overline{D(A)}) \times C_b([0, +\infty); \overline{D(B)}) \to C([0, 0]; X)$ and $q: C_b([0, +\infty); \overline{D(A)}) \times C_b([0, +\infty); \overline{D(B)}) \to C([0, 0]; Y)$ are non-expansive functions with affine growth.

Partial differential equations with nonlocal initial conditions arise in many areas of applied mathematics and represent mathematical models of various phenomena. See Deng [18] and McKibben [25]. The study for nonlocal Cauchy problems without delay was initiated by Byszewski [15] (in the semilinear case), and subsequently it has been developed by many authors. We mention here some significant contributions to the field: Aizicovici and Lee [1], Aizicovici and McKibben [2], García-Falset [21], García-Falset and Reich [22], Cardinali, Precup and Rubbiioni [16] in the single-valued case, Aizicovici and Staicu [3], Paicu and Vrabie [32], Zhu and Li [43] in the multi-valued case. Nica [31] proved the existence of the solutions for nonlinear first order differential systems with nonlocal conditions. These results were extended by Bolotan-Nica, Infante and Precup [7] to differential systems with nonlinear and nonlocal boundary conditions. For delay evolution equations with local initial conditions see Mitidieri and Vrabie [26], [27], Necula and Popescu [28], and the references therein. As far as nonlocal initial conditions are concerned, we mention the papers Burlică and Roşu [11], Burlică, Roşu and Vrabie [13], Necula, Popescu and Vrabie [29], Vrabie [37]-[41], Wang and Zhu [42]. For parabolic systems with nonlinear, nonlocal initial conditions we mention the paper of Infante and Maciøjewski [24]. Concerning the reaction-diffusion systems without delay see: Burlică [8], Burlică and Roşu [9], [10], Díaz and Vrabie [19], Necula and Vrabie [30], Roşu [33], [34]. Existence results for reaction-diffusion systems with delay and nonlocal initial conditions were obtained in Burlică, Roşu and Vrabie [14] for the single-valued case and by Burlică and Roşu [12] for the multi-valued case. The present work complements Burlică, Roşu and Vrabie [14] by allowing the nonlocal initial constraint function $p$ to have affine instead of linear growth with respect to the first argument and $q$ to obey the same property with respect to its second variable. Moreover, we allow the unknown functions to have different delays, $\tau_1$ and $\tau_2$. Our general assumptions include reaction-diffusion systems in which one or both