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MOTION PLANNING ALGORITHMS FOR CONFIGURATION SPACES IN THE HIGHER DIMENSIONAL CASE

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Abstract. The aim of this paper is to give an explicit motion planning algorithm for configuration spaces in the higher dimensional case.

1. Introduction

The topological approach to the motion planning problem was introduced by Farber in [2] and [3]. A motion planning problem is a rule assigning a continuous path to given two configurations – initial point and desired final point of a robot. Farber introduced the notion of topological complexity which measures the discontinuity of any motion planner in a configuration space. In [6], Rudyak introduced higher topological complexity, the concept fully developed in [1]. Higher topological complexity is related to motion planning problem which assigns a continuous path (with n-legs) to given n configurations. More precisely, it can be understood as a motion planning algorithm when a robot travels from the initial point A_1 to A_2, then from A_2 to A_3, and this keeps going until it reaches at the desired final point A_n.

This paper is based on the work of Mas–Ku and Torres–Giese who gave an explicit motion planning algorithm for configuration spaces $F(\mathbb{R}^2, k)$ and $F(\mathbb{R}^n, k)$, in [5]. In the last section, we will consider the higher dimensional case

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in the sense of Rudyak in [6], and give an explicit motion planning algorithm for this case.

2. Preliminaries

In this section, we will re-phrase the definitions and propositions for $F(\mathbb{R}^n, k)$ which are given in [5].

A vector $A = (a_1, \dots, a_l)$ (where a_i is a positive integer for $i = 1, \dots, l$) which satisfies $\sum a_i = k$ is called a partition of k. Here, the number $|A| = l$ is called the number of levels of A.

Recall the reverse lexicographic order on $\mathbb{R}^n$: $(b_1, \dots, b_n) \le (c_1, \dots, c_n)$ if there is an index $k \in \{1, \dots, n\}$ such that $b_i = c_i$ for $k < i \le n$ and $b_k < c_k$.

As stated in [5], if $x = (x_1, \dots, x_k) \in F(\mathbb{R}^2, k)$, then there is a unique permutation $\sigma \in \Sigma_k$ such that $x_{\sigma(1)} < \dots < x_{\sigma(k)}$. Such a permutation is denoted by σ_x. A similar argument can be stated for $F(\mathbb{R}^n, k)$, namely, if $x = (x_1, \dots, x_k) \in F(\mathbb{R}^n, k)$, then there is a unique permutation $\sigma \in \Sigma_k$ such that $x_{\sigma(1)} < \dots < x_{\sigma(k)}$.

Let $\pi_n \colon \mathbb{R}^n \to \mathbb{R}$, given by $\pi_n(x_1, \dots, x_n) = x_n$, be the projection to the n-th factor. For the configuration $x = (x_1, \dots, x_k) \in F(\mathbb{R}^n, k)$ which is reverse lexicographically ordered, we can find positive integers $a_1, \dots, a_l$ as follows:

$$\begin{aligned}
\pi_n(x_1) &= \dots = \pi_n(x_{a_1}) < \pi_n(x_{a_1+1}), \\
\pi_n(x_{a_1+1}) &= \dots = \pi_n(x_{a_1+a_2}) < \pi_n(x_{a_1+a_2+1}), \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
\pi_n(x_{a_1+\dots+a_{l-2}+1}) &= \dots = \pi_n(x_{a_1+\dots+a_{l-1}}) < \pi_n(x_{a_1+\dots+a_{l-1}+1}), \\
\pi_n(x_{a_1+\dots+a_{l-1}+1}) &= \dots = \pi_n(x_{a_1+\dots+a_l}) = \pi_n(x_k).
\end{aligned}$$

Since $a_1 + \dots + a_l = k$, $(a_1, \dots, a_l)$ is a partition of k. This partition is denoted by A_x. If A is obtained from the configuration x as in the above paragraph, then x is called an A-configuration.

Let $x = (x_1, \dots, x_k) \in F(\mathbb{R}^n, k)$ be an A-configuration. Then x has $|A|$ levels. Moreover, x_i and x_j are said to have the same level if $\pi_n(x_i) = \pi_n(x_j)$. Given a partition A of k and a permutation $\sigma \in \Sigma_k$, let

$$F_{A,\sigma} = \{x = (x_1, \dots, x_k) \in F(\mathbb{R}^n, k) : \sigma_x = \sigma \text{ and } x \text{ is an } A\text{-configuration}\}.$$

Define

$$F_A = \bigcup_{\sigma \in \Sigma_k} F_{A,\sigma}.$$

In fact, F_A denotes the set consisting of configurations x which produce A. Moreover, notice that $F(\mathbb{R}^n, k) = \bigcup\limits_{A} F_A$.