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MULTIPLE SOLUTIONS WITH PRESCRIBED MINIMAL PERIOD FOR SECOND ORDER ODD NEWTONIAN SYSTEMS WITH SYMMETRIES

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Abstract. For an orthogonal Γ-representation V (Γ is a finite group) and for an even Γ-invariant C^2-functional $f\colon V \to \mathbb{R}$ satisfying the condition $0 < \theta\nabla f(x) \bullet x \leq \nabla^2 f(x)x \bullet x$ (for $\theta > 1$ and $x \in V \setminus \{0\}$), we consider the odd Newtonian system $\ddot{x}(t) = -\nabla f(x(t))$ and establish the existence of multiple periodic solutions with a minimal period p (for any given $p > 0$). As an example, we prove the existence of arbitrarily many periodic solutions with minimal period p for a specific D_n-symmetric Newtonian system.

1. Introduction

The purpose of this paper is to study the existence of multiple periodic solutions with a given minimal period $p > 0$ for the Newtonian systems of the

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type

$$\ddot{x} = -\nabla f(x), \quad x \in V := \mathbb{R}^n, \tag{1.1}$$

where V is an orthogonal Γ-representation (with Γ being a finite group), $f\colon V \to \mathbb{R}$ is a Γ-invariant even C^2-function satisfying the condition

$$0 < \theta \nabla f(x) \bullet x \leq \nabla^2 f(x) x \bullet x \quad \text{for } x \in V \setminus \{0\}, \tag{1.2}$$

for some $\theta > 1$.

The problem of finding periodic solutions with minimal period for variational problems is not new and there is a large literature devoted to this topic. Beginning with the pioneering work of P. Rabinowitz (cf. [21]), the question of existence of such solutions in the first order Hamiltonian systems

$$\frac{dz}{dt} = J\nabla f(z), \qquad J = \begin{bmatrix} 0 & -\mathrm{Id} \\ \mathrm{Id} & 0 \end{bmatrix}, \tag{1.3}$$

was discussed in the works of Clark and Ekeland (cf. [6]), Ambrosetti and Mancini (cf. [2]), Girardi and Matzeu (cf. [13], [14]), Deng (cf. [7]), Ekeland and Hofer (cf. [9]), Zhang and Tang (cf. [23], 2013), Michalek and Tarantello (cf. [20]), Fei et al. (cf. [12], [10]), Liu and Wang (cf. [19]), and many others. These authors used various methods, such as the Mountain Pass lemma, finite-dimensional approximations, duality principle, index theory, and restrictions to Nehari manifold, to prove several interesting existence results for multiple periodic solutions with the prescribed minimal period.

On the other hand, although there are many existence results for multiple periodic solutions of (1.3) with prescribed minimal period, there are only few such results for system (1.1). The existence of such periodic solutions was discussed, for example, in papers by Long (cf. [16, 17]), Fei et al. (cf. [11]), Xiao (cf. [22]), and others. The main goal of this paper is to combine the Nehari manifold techniques with the H-fixed-point reduction method in order to show the existence of multiple periodic solutions with the prescribed minimal period for symmetric Newtonian systems satisfying condition (1.2). More precisely, by exploring Γ-symmetries of system (1.1), where Γ is a finite group, and by applying the H-fixed-point reduction (for a specific subgroup $H \subset \Gamma \times \mathbb{Z}_2 \times S^1$), we show that the second order odd Newtonian system (1.1), where V is a Γ-representation and f is a Γ-invariant function (Γ is a finite group), has multiple p-periodic solutions with the minimal period p. Depending on the size of the group Γ, this number of p-periodic solutions may be arbitrarily large. Our approach is based on the usage of the Nehari manifold techniques developed by Yu Ming Xiao in [22].

Our main result, Theorem 4.4, mainly states that for any Γ-symmetric Newtonian system (1.1), satisfying (1.2), and for any $p > 0$, there exist multiple