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WELL-POSEDNESS FOR MIXED QUASI-VARIATIONAL-HEMIVARIATIONAL INEQUALITIES

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Abstract. In this paper, we consider the well-posedness of mixed quasi-variational-hemivariational inequalities ((MQVHVI) for short). By introducing a new concept of the α-η-monotone mappings, we establish several metric characterizations and equivalent conditions of well-posedness for (MQVHVI).

1. Introduction

As an important and useful generalization of variational inequalities, the theory of hemivariational inequalities was firstly introduced by P.D. Panagiotopoulos (cf. [30], [32]–[34]) as variational expressions for several classes of mechanical problems with nonsmooth and nonconvex energy superpotentials. Because of their important applications in mechanics and engineering, especially in nonsmooth analysis and optimization, hemivariational inequalities have extensively been studied by many authors recently. For more related works, we refer to [3], [5]–[7], [13], [18]–[26], [29], [30], [35], [40], [42] and the references therein.

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On the other hand, the classical concept of well-posedness for minimization problems, which has been known as the Tykhonov well-posedness, is due to Tykhonov [37], which requires the existence and uniqueness of solution to global minimization problems and the convergence of every minimizing sequence toward the unique solution. However, in many practical situations, the solution may be not unique for an optimization problem. Thus, the concept of well-posedness in the generalized sense was introduced, which means the existence of solutions and the convergence of some subsequence of every minimizing sequence toward a solution. The researched topic is important, because the well-posedness of the problems plays a crucial role in numerical analysis and there is a need to study the convergence of approximating sequences. So, many authors were devoted to generalizing the concept of well-posedness of optimization problems (see [1], [10]), variational inequalities (see [9], [11], [12], [27], [36]), fixed point problems (see [17]), equilibrium problems (see [14], [16], [38], [39], [28]), inclusion problems (see [8]), etc.

However, there are very few results on well-posedness for hemivariational inequalities. Goeleven and Mentagui [13] firstly generalized the well-posedness concept to hemivariational inequalities and presented some basic results concerning the well-posed hemivariational inequalities. Recently Xiao et al. in [40], and [41] considered the well-posedness for a class of variational-hemivariational inequalities and obtained some equivalence results for well-posedness of hemivariational inequalities. They gave some metric characterizations for the well-posed variational-hemivariational inequalities.

Motivated by the aforementioned works, we shall investigate the well-posedness of mixed quasi-variational-hemivariational inequalities. We establish several metric characterizations and equivalent conditions of well-posedness for mixed quasi-variational-hemivariational inequalities.

In the sequel, let K be a nonempty, closed and convex subset of a real Banach space E with its dual E^*, $F\colon K \rightrightarrows E^*$ and $S\colon K \rightrightarrows K$ be two set-valued mappings. Let $T\colon K \to E^*$ be a perturbation term, $\phi\colon K \to \mathbb{R}\cup\{+\infty\}$ be a proper convex functional, $\eta\colon E\times E \to E$ and $f \in E^*$. In this paper, we shall deal with the following mixed quasi-variational-hemivariational inequality ((MQVHVI) for short):

Find $u_0 \in S(u_0)$ and $u_0^* \in F(u_0)$ such that

$$\langle u_0^*, \eta(v,u_0)\rangle + \langle Tu_0 - f, v-u_0\rangle + J^\circ(u_0; v-u_0) + \phi(v) - \phi(u_0) \geq 0,$$

for all $v \in S(u_0)$, where $J^\circ(u;v)$ denotes the generalized directional derivative of a locally Lipschitz functional $J\colon E \to \mathbb{R}$ at u in the direction v.

Now, let us consider some special cases of problem (MQVHVI).

Case 1. If $\eta(v,u) = v-u$, $T=0$, $\phi=0$, then (MQVHVI) is reduced to the following generalized quasi-variational-hemivariational inequality: