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THERMO-VISCO-ELASTICITY FOR MODELS WITH GROWTH CONDITIONS IN ORLICZ SPACES

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Abstract. We study a quasi-static evolution of the thermo-visco-elastic model. We act with external forces on a non-homogeneous material body, which is a subject of our research. Such action may cause deformation of this body and may change its temperature. Mechanical part of the model contains two kinds of deformation: elastic and visco-elastic. The mechanical deformation is coupled with temperature and both of them may influence each other. Since the constitutive function on evolution of the visco-elastic deformation depends on temperature, the visco-elastic properties of material also depend on temperature. We consider the thermodynamically complete model related to a hardening rule with growth condition in generalized Orlicz spaces. We provide the proof of existence of solutions for such class of models.

1. Introduction

The objective of this paper is to show the existence of solutions to a special class of thermo-visco-elastic models. We consider the reaction of a material body treated by external forces and heat flux through the boundary. In the case of

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ideal elastic deformations, the body should return to its initial state after termination of external forces activity. However, if deformations are not elastic, i.e. there is a loss of potential energy, we deal with a special kind of inelastic deformations. The potential energy lost during the process may be transformed into the thermal energy. We focus on the visco-elastic type of deformations, which for instance may be observed in polymers. Both deformations are coupled in physical phenomena and they may be observed at the same time. Consequently, these two types of deformations appear in the models considered in this paper. The elastic deformation is reversible, whereas the visco-elastic one is irreversible.

The thermo-visco-elastic system of equations, as a consequence of balance of momentum and balance of energy, cf. [19], [33], see also [21], captures displacement, temperature and visco-elastic strain. Since these two principles do not take into account the material properties of considered body, we may complement it by adding constitutive relations which complete missing information. The standard technique in the solid body deformation is to work with two constitutive relations. The first one describes the dependency between stress and strains, i.e. this is an equation for the Cauchy stress tensor. The second one is a constitutive equation which is characterized by the evolution of visco-elastic strain tensor.

We assume that the body $\Omega \subset \mathbb{R}^3$ is an open bounded set with a C^2 boundary. Then the quasi-static evolution problem is formulated by the following system of equations:

$$
\begin{cases}
-\operatorname{div} \boldsymbol{T} = \boldsymbol{f} & \text{in } \Omega \times (0,T),\\
\boldsymbol{T} = \boldsymbol{D}(\boldsymbol{\varepsilon}(\boldsymbol{u}) - \boldsymbol{\varepsilon}^{\mathbf{p}}) & \text{in } \Omega \times (0,T),\\
\boldsymbol{\varepsilon}^{\mathbf{p}}_t = \boldsymbol{G}(\theta, \boldsymbol{T}^d) & \text{in } \Omega \times (0,T),\\
\theta_t - \Delta\theta = \boldsymbol{T}^d : \boldsymbol{G}(\theta, \boldsymbol{T}^d) & \text{in } \Omega \times (0,T).
\end{cases}
\tag{1.1}
$$

By the solution of this system we understand finding the displacement of material $\boldsymbol{u}\colon \Omega \times \mathbb{R}_+ \to \mathbb{R}^3$, the temperature of material $\theta\colon \Omega \times \mathbb{R}_+ \to \mathbb{R}_+$ and the visco-elastic strain tensor $\boldsymbol{\varepsilon}^{\mathbf{p}}\colon \Omega \times \mathbb{R}_+ \to \mathcal{S}^3_d$. We denote by $\mathcal{S}^3$ the set of symmetric 3×3-matrices with real entries and by $\mathcal{S}^3_d$ a subset of $\mathcal{S}^3$ which contains traceless matrices. The function $\boldsymbol{T}\colon \Omega \times \mathbb{R}_+ \to \mathcal{S}^3$ stays for the Cauchy stress tensor. By $\boldsymbol{I}$ we mean the identity matrix from $\mathcal{S}^3$, thus $\boldsymbol{T}^d$ is the deviatoric (traceless) part of the tensor $\boldsymbol{T}$, i.e. $\boldsymbol{T}^d = \boldsymbol{T} - \operatorname{tr}(\boldsymbol{T})\boldsymbol{I}/3$. Additionally, we denote by $\boldsymbol{\varepsilon}(\boldsymbol{u})$ the deformation tensor associated to $\boldsymbol{u}$, i.e. $\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u})/2$.

The motivation for the current paper is to extend results presented in [20], where we proved the existence of solutions to the Norton–Hoff model, i.e. the model with the growth condition on the visco-elastic strain tensor in Lebesgue spaces. The model with the growth condition in generalized Orlicz spaces is a