FRACTIONAL ORDER SEMILINEAR VOLterra INTEGRODIFFERENTIAL EQUATIONS IN BANACH SPACES

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ABSTRACT. Sufficient conditions are established for the existence results of fractional order semilinear Volterra integrodifferential equations in Banach spaces. Results are obtained by using the theory of fractional cosine families and fractional powers of operators.

1. Introduction


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In recent years, fractional differential equations have received increasing attention due to its applications in physics, chemistry, materials, engineering, biology, finance, we refer to [19], [13], [15]. Fractional order derivatives have the memory property and can describe many phenomena that integer order derivatives cannot characterize.

Consider the following fractional semilinear differential equation:

\[(1.1) \begin{cases} C^\alpha_D u(t) = Au(t) & \text{for } t > 0, \\ u(0) = x, u^{(k)}(0) = 0 & \text{for } k = 1, \ldots, m-1, \end{cases} \]

where $\alpha > 0$, $m$ is the smallest integer greater than or equal to $\alpha$, $C^\alpha_D$ is the $\alpha$-order Caputo fractional derivative operator, $A : D(A) \subset X \to X$ is a closed densely defined linear operator on a Banach space $X$.

Bazhlekoova [1] introduced the notion of solution operator for (1.1) as follows.

**Definition 1.1.** A family $\{C_\alpha(t)\}_{t \geq 0} \subset B(X)$ is called a solution operator for (1.1) if the following conditions are satisfied:

(a) $C_\alpha(t)$ is strongly continuous for $t \geq 0$ and $C_\alpha(0) = I$ (the identity operator on $X$);

(b) $C_\alpha(t)D(A) \subset D(A)$ and $AC_\alpha(t)\xi = C_\alpha(t)A\xi$ for all $\xi \in D(A)$, $t \geq 0$;

(c) $C_\alpha(t)\xi$ is a solution of $x(t) = \xi + \int_0^t g_\alpha(t-s)Ax(s)\,ds$ for all $\xi \in D(A)$, $t \geq 0$, we refer to equality (2.3) concerning the definition of $g_\alpha(t)$.

$A$ is called the infinitesimal generator of $C_\alpha(t)$. Note that in some literature the solution operator also is called the fractional resolvent family or fractional resolvent operator function, see [2], [11]. As a matter of fact, the solution operator $C_2(t)$ is a cosine family, in this paper, for $\alpha \in (1, 2]$, the solution operator