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FRACTIONAL ORDER SEMILINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. Sufficient conditions are established for the existence results of fractional order semilinear Volterra integrodifferential equations in Banach spaces. Results are obtained by using the theory of fractional cosine families and fractional powers of operators.

1. Introduction

The integrodifferential equations in Banach spaces have attracted much interest. Prüss [20] considered the solvability behavior on the real line of linear integrodifferential equations in a general Banach space and gave several applications to integral partial differential equations. Grimmer [5] established general conditions to ensure the existence of a resolvent operator for an integrodifferential equation in Banach spaces. Fitzgibbon [4] studied the existence, continuation, and behavior of solutions to an abstract semilinear Volterra integrodifferential equation. Keyantuo and Lizama [8] characterized existence and uniqueness of solutions for a linear integrodifferential equation in Hölder spaces. Londen [12] proved an existence result on a nonlinear Volterra integrodifferential equation in real reflexive Banach spaces by using the theory of maximal monotone operators. Prüss [22] studied linear Volterra integrodifferential equations in Banach

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spaces in case the main part of the equation generates an analytic C_0-semigroup. Travis and Webb [23] studied the existence of solutions to semilinear second order Volterra integrodifferential equations in Banach spaces by using the theory of strongly continuous cosine families. Mainini and Mola [14] considered in an abstract setting, an instance of the Coleman–Gurtin model for heat conduction with memory. Engler [3] constructed global weak solution of scalar second-order quasilinear hyperbolic integrodifferential equations with singular kernels. Prüss [21] studied the existence, positivity, regularity, compactness and integrability of the resolvent for a class of Volterra equations of scalar type. Hernández [6] studied the existence of strict and classical solutions for a class of abstract nonautonomous Volterra integrodifferential equations in Banach spaces. Lang and Chang [10] investigated the local existence and uniqueness of solutions to integrodifferential equations with infinite delay. Jawahdou [7] studied the existence of mild solutions for initial value problems for semilinear Volterra integrodifferential equations in Banach spaces.

In recent years, fractional differential equations have received increasing attention due to its applications in physics, chemistry, materials, engineering, biology, finance, we refer to [19], [13], [15]. Fractional order derivatives have the memory property and can describe many phenomena that integer order derivatives cannot characterize.

Consider the following fractional semilinear differential equation:

$$\begin{cases} {}^C D_t^\alpha u(t) = Au(t) & \text{for } t > 0, \\ u(0) = x, u^{(k)}(0) = 0 & \text{for } k = 1, \ldots, m-1, \end{cases} \tag{1.1}$$

where $\alpha > 0$, m is the smallest integer greater than or equal to α, ${}^C D_t^\alpha$ is the α-order Caputo fractional derivative operator, $A\colon D(A) \subset X \to X$ is a closed densely defined linear operator on a Banach space X.

Bazhlekova [1] introduced the notion of solution operator for (1.1) as follows.

Definition 1.1. A family $\{C_\alpha(t)\}_{t\geq 0} \subset \mathcal{B}(X)$ is called a *solution operator* for (1.1) if the following conditions are satisfied:

(a) $C_\alpha(t)$ is strongly continuous for $t \geq 0$ and $C_\alpha(0) = I$ (the identity operator on X);
(b) $C_\alpha(t)D(A) \subset D(A)$ and $AC_\alpha(t)\xi = C_\alpha(t)A\xi$ for all $\xi \in D(A)$, $t \geq 0$;
(c) $C_\alpha(t)\xi$ is a solution of $x(t) = \xi + \int_0^t g_\alpha(t-s)Ax(s)\,ds$ for all $\xi \in D(A)$, $t \geq 0$, we refer to equality (2.3) concerning the definition of $g_\alpha(t)$.

A is called the *infinitesimal generator* of $C_\alpha(t)$. Note that in some literature the solution operator also is called the fractional resolvent family or fractional resolvent operator function, see [2], [11]. As a matter of fact, the solution operator $C_2(t)$ is a cosine family, in this paper, for $\alpha \in (1,2]$, the solution operator