**T**opological **M**ethods in **N**onlinear **A**nalysis
Volume 47, No. 1, 2016, 241–264
DOI: 10.12775/TMNA.2016.004



# EXISTENCE AND ASYMPTOTIC BEHAVIOUR OF GROUND STATE SOLUTION FOR QUASILINEAR SCHRÖDINGER–POISSON SYSTEMS IN $\mathbb{R}^3$

Ling Ding — Lin Li — Yi-Jie Meng — Chang-Ling Zhuang

Abstract. In this paper, we are concerned with existence and asymptotic behavior of ground state in the whole space $\mathbb{R}^3$ for quasilinear Schrödinger–Poisson systems

$$\begin{cases} -\Delta u + u + K(x)\phi(x)u = a(x)f(u), & x \in \mathbb{R}^3, \\ -\mathrm{div}[(1+\varepsilon^4|\nabla\phi|^2)\nabla\phi] = K(x)u^2, & x \in \mathbb{R}^3, \end{cases}$$

when the nonlinearity coefficient $\varepsilon > 0$ goes to zero, where $f(t)$ is asymptotically linear with respect to $t$ at infinity. Under appropriate assumptions on $K$, $a$ and $f$, we establish existence of a ground state solution $(u_\varepsilon, \phi_{\varepsilon,K}(u_\varepsilon))$ of the above system. Furthermore, for all $\varepsilon$ sufficiently small, we show that $(u_\varepsilon, \phi_{\varepsilon,K}(u_\varepsilon))$ converges to $(u_0, \phi_{0,K}(u_0))$ which is the solution of the corresponding system for $\varepsilon = 0$.

## 1. Introduction and main results

Consider the following quasilinear Schrödinger–Poisson systems

$$\begin{cases} -\Delta u + u + K(x)\phi(x)u = a(x)f(u), & x \in \mathbb{R}^3, \\ -\mathrm{div}[(1+\varepsilon^4|\nabla\phi|^2)\nabla\phi] = K(x)u^2, & x \in \mathbb{R}^3, \end{cases} \tag{1.1}$$

2010 *Mathematics Subject Classification.* Primary: 35Q55, 82D99; Secondary: 35B25, 35B40, 35J50, 47J30, 58E05, 81Q05.

*Key words and phrases.* Schrödinger–Poisson system; variational methods; ground state solution; asymptotically linear; asymptotic behavior.

The first author was supported by the Key Project in Science and Technology Research Plan of the Education Department of Hubei Province (No. D20142602).

where $K \in L^2(\mathbb{R}^3)$, $K \geq (\not\equiv)\, 0$, $a$ is a positive bounded function, and $f \in C(\mathbb{R}, \mathbb{R}^+)$. When $\varepsilon = 0$, this Schrödinger–Poisson system arises in an interesting physical model which describes the interaction of a charged particle with electromagnetic field (see [3] and the references therein). When $\varepsilon \neq 0$, system (1.1) firstly arises this form like

$$\begin{cases} i\partial_t u = -\dfrac{1}{2}\Delta u + (V + \phi(x))u, & x \in \mathbb{R}^3, \\ -\operatorname{div}[\varepsilon(\nabla\phi)\nabla\phi] = |u|^2 - n^*, & x \in \mathbb{R}^3, \\ u(x,0) = u(x), & x \in \mathbb{R}^3, \end{cases}$$

which corresponds to a quantum mechanical model where the quantum effects are important, as in the case of microstructures (see for example Markowich, Ringhofer and Schmeiser [22]). The charge density $n(x,t)$ derives from the Schrödinger wave function $u(x,t)$ by $n(x,t) = |u(x,t)|^2$, while $n^*$ and $V$ represent respectively a dopant-density and a real effective potential which are time-independent. More details dealing with the phenomenon may be found in [17], [18] and references therein. After that, in [1], [16], that the field dependent dielectric constant in Poisson equation has the form

$$\varepsilon(\nabla\phi) = \varepsilon^0 + \varepsilon^1|\nabla\varphi|^2, \quad \varepsilon^0, \varepsilon^1 > 0.$$

Existence and uniqueness of global strong solutions and existence results of solutions of the form $u(x,t) = e^{i\omega t}u(x)(\omega,\ u(x) \in \mathbb{R})$ are obtained under suitable conditions, respectively. Moreover, in [4], authors obtained that the existence of standing waves (actually ground states) solutions for the Schrödinger–Poisson system with $\varepsilon^0 = 1$ and $\varepsilon^1 = \varepsilon^4$ of

$$\begin{cases} -\dfrac{1}{2}\Delta u + (V + \phi(x))u = 0, & x \in \mathbb{R}^3, \\ -\operatorname{div}[(1 + \varepsilon^4|\nabla\phi|^2)\nabla\phi] = |u|^2 - n^*, & x \in \mathbb{R}^3 \end{cases} \tag{1.2}$$

and with their asymptotic behavior when the nonlinearity coefficient in the Poisson equation $\varepsilon$ goes to zero with suitable potential $V$.

From the mathematical view, if the Schrödinger equation with only one nonlinear nonlocal term $\phi(x)u$ in system (1.2) is replaced by the other different version of Schrödinger equations which have other nonlinear terms besides the nonlinear nonlocal term, we want to know that whether ground state solutions exist and if exists whether they converge to ones of the corresponding system for $\varepsilon = 0$. In this paper, we shall answer these questions about system (1.1).

When $\phi \equiv 0$, system (1.1) becomes into a single equation

$$-\Delta u + u = a(x)f(u). \tag{1.3}$$