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EXISTENCE RESULTS FOR A CLASS OF HEMIVARIATIONAL INEQUALITIES INVOLVING THE STABLE (g, f, α)-QUASIMONOTONICITY

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Abstract. In this paper, by introducing a new concept of the stable (g, f, α)-quasimonotonicity and applying the properties of Clarke's generalized gradient and KKM technique, we show the existence results of solutions for hemivariational inequalities when the constraint set is compact, bounded and unbounded, respectively, which extends and improves several well-known results in many respects. In the last section, we also give an example to present the our main result.

1. Introduction

As an important and useful generalization of variational inequalities, hemivariational inequalities were first introduced by Panagiotopoulos (see [15], [16]) as the variational formulation of an important class of unilateral or inequality problems in mechanics. It is based on the notation of Clarke's generalized gradient for a class of locally Lipschitz functions. Hemivariational inequalities appear in a variety of mechanical problems, for example, the unilateral contact problems

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in nonlinear elasticity, the problems describing the adhesive and friction effects, the nonconvex semipermeability problems, the masonry structures, and the delamination problems in multilayered composites (one can see [13], [14], [17]). In the last few years many kinds of hemivariational inequalities have been studied (see [2], [3], [9]–[12], [20]) and the study of hemivariational inequalities has emerged as a new and interesting branch of applied mathematics.

Very recently, many authors studied the existence results for some types of hemivariational inequalities (see [18], [19], [21]). In 2011, Zhang and He ([21]) study a kind of hemivariational inequalities of the Hartman–Stampacchia type by introducing the concept of stable quasimonotonicity. They considered that the constraint set is a bounded (or unbounded), closed and convex subset in a reflexive Banach space. The authors gave sufficient conditions for the existence and boundedness of solutions. In 2013, Tang and Huang ([18]) generalized the result of [21], by introducing the concept of stable ϕ-quasimonotonicity. By applying the stable ϕ-quasimonotonicity and the properties of Clarke's generalized directional derivative and generalized gradient, they obtained some existence theorems when the constrained set is nonempty, bounded (or unbounded), closed and convex in a reflexive Banach space. In the same year, Wangkeeree and Preechasilp ([19]) generalized the results of [18] and [21], by introducing the concept of stable f-quasimonotonicity. By applying the stable f-quasimonotonicity, they obtained some existence theorems similar to [18].

The aim of this paper is to study the existence of solutions for generalized problems of hemivariational inequalities in a reflexive Banach space. To establish our results, we introduce a new concept of stable (g, f, α)-quasimonotonicity and use the properties of Clarke's generalized directional derivative, generalized gradient, and KKM technique. Our results extend and improve some results in [18], [19], [21] in many respects.

The rest of this paper is organized as follows. In the next section, we will introduce some useful preliminaries and necessary materials. In Section 3, we introduce some kinds of generalized monotonicity of a mapping. In Section 4, we are devoted to proving our main results. We show the existence of solutions in the case when the constraint set is compact, bounded and unbounded in Theorems 4.1, 4.2 and 4.6, respectively. Theorem 4.8 provides a sufficient condition to the boundedness of the solution set. In Section 5, we give an example to present the generalized monotonicity and our main result.

2. Preliminaries

Let E be a real Banach space with the norm denoted by $\|\cdot\|_E$. Denote by E^* its dual space and by $\langle\cdot\,,\cdot\rangle_E$ the duality pairing between E^* and E. Let $F\colon K(\subseteq E) \rightrightarrows E^*$ be a multivalued mapping, $g\colon K\times K \to E$ be a mapping.