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HOMOTOPY PERTURBATION METHOD FOR TWO POINT BOUNDARY VALUE PROBLEMS

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Abstract. The homotopy perturbation method is applied for solving two point boundary value problems. In this method a trial function (initial solution) is chosen with some unknown parameters, which are identified using the method of weighted residuals. An example is given, the obtained result is compared with the exact solution, revealing that this method is very efficient and the obtained solution is of high accuracy.

1. Introduction

Some of the most common problems in applied sciences and engineering are usually formulated as two point boundary value problems. A well known fact is that exact solutions in closed form of such problems do not exist in many cases. This fact makes approximate solutions of special interest.

In this paper, we will consider a nonlinear two point boundary value problem of the form [1]:

$$y'' - \frac{3}{2}y^2 = 0, \quad 0 \le x \le 1 \tag{1.1}$$

Subject to

$$y(0) = 4, \quad y(1) = 1.$$

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The problem was studied by Lesnic using the Adomian method [10]. As pointed out by many authors [1], [10] that it is very complex and time consuming to calculate the Adomiam polynomial. Recently, various different analytical methods were applied to nonlinear equations arising in engineering applications, such as the Adomian method [1], [10], the variational iteration method [2], [8], [13], [15]–[17], the homotopy perturbation method [3]–[6], [14], exp-function method [18], [19], and variational methods [9], [11], [12], a complete review is available in [7]. In this paper, the homotopy perturbation method [5], [7] is applied to the discussed problem, and the obtained results show that the method is very effective and simple.

2. Basic idea of He's homotopy perturbation method

In order to use the homotopy perturbation, we construct a homotopy in the form [5], [7]

$$(1-p)(y''-y_0^2)+p\left(y''-\frac{3}{2}y^2\right)=0, \tag{3.1}$$

with initial approximation

$$y_0(x)=ax^2-(4+a)x+4 \tag{3.2}$$

where a is an unknown constant to be further determined. It is obvious that equation (3.2) satisfies the boundary conditions.

We re-write equation (3.1) in the form

$$y''+2a+p\left(\frac{3}{2}y^2-2a\right)=0. \tag{3.3}$$

We suppose the solution of (3.3) has the form:

$$y(x)=y_0(x)+py_1(x)+p^2y_2(x)+\dots. \tag{3.4}$$

Substituting (3.4) into (1.1) and equating the terms with the identical powers of p , we can solve $y_0, y_1, y_2, \cdots$ sequentially with ease. Setting $p=1$, we obtain the approximate solution of (1.1) in the form:

$$y(x)=y_0(x)+y_1(x)+y_2(x)+\dots.$$

We can easily obtain sequentially

$$y_0''=2a, \quad y_1''=-2a+\frac{3}{2}y_0^2, \quad y_2''=-3y_0y_1.$$

We, therefore, obtain the second-order approximate solution in the form

$$\begin{aligned} y(x)=\frac{1}{20}a^2x^6-\left(\frac{3}{20}a^2+\frac{9}{20}a\right)x^5+\left(\frac{1}{8}a^2+\frac{7}{4}a+\frac{9}{8}\right)x^4 \\ -(6+2a)x^3+12x^2-\left(\frac{1}{40}a^2-\frac{7}{10}a+\frac{81}{8}\right)x+4. \end{aligned} \tag{3.5}$$

In order to identify the unknown constant a in (3.5), we apply the method of weighted residuals. Substituting (3.5) into (1.1) results in the following residual

$$R(x,a) = y'' - \frac{3}{2}y^2.$$

Using the least squares method, we have

$$\int_0^1 R\frac{\partial R}{\partial a}\,dx = 0,$$

yielding the result $a = 3.25$. Figure 1 shows the remarkable accuracy of the obtained result.

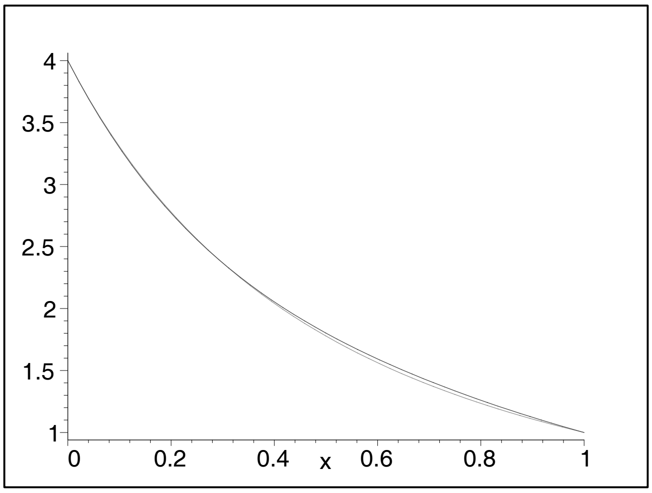

Figure 1. Comparison of the obtained result (continuous line) with exact solution (discontinuous line)

3. Conclusions

In this paper, we present the homotopy perturbation method for solving two point boundary value problems. The homotopy perturbation method is of remarkable simplicity, while the obtained results are of utter accuracy. The method can be applied to various other nonlinear problems without any difficulty.

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