Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 31, 2008, 359–367

SOLITARY WAVE SOLUTIONS FOR A COUPLED MKDV SYSTEM USING THE HOMOTOPY PERTURBATION METHOD

Yong-Qing Jiang — Jia-Min Zhu

Abstract. The work presents a derivation of solitary solutions of a coupled MKdV system using the homotopy perturbation method.

1. Introduction

In this paper, we consider a coupled MKdV equation which was introduced by Wu et al. [12]. In [12], the authors introduced a 4×4 matrix spectral problem with three potentials and proposed a corresponding hierarchy of nonlinear equations. One of the typical equations in the hierarchy is a new coupled MKdV system which can be written as follows:

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3u^2u_x + \frac{3}{2}v_{xx} + 3(uv)_x - 3\lambda u_x, \\ v_t &= -v_{xxx} + 3vv_x + 3v_xu_x - 3u^2v_x - 3\lambda v_x. \end{aligned} \tag{1.1}$$

The system becomes a generalized KdV equation when $u = 0$ and a MKdV equation when $v = 0$, respectively. More recently, the soliton solutions for this system were constructed by Fan [3]. The discussed coupled MKdV system was

2000 *Mathematics Subject Classification.* Primary 65C20, 34E10; Secondary 34E05.

Key words and phrases. Homotopy perturbation method, coupled MKdV equation, solitary wave solution, approximations solution.

This work was supported in part by NSF of China under the Grant NO. 10172056.

also studied by many authors via different approaches, for example, trigonometric function transform method by Cao et al. [2], the extended tanh-function method by Fan [3], Jacobian elliptic function method by Hassan et al. [6], and Adomian's decomposition method by Raslan [11].

The homotopy perturbation method (HPM) was first proposed by He [7], [8]. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$ which is considered as a "small parameter". Recently, many researchers did a lot of significant work about the homotopy perturbation method [1], [4], [5].

In this paper, we further extend the method to solve the coupled MKdV system. Using HPM, we get some explicit solutions of the coupled MKdV system without using any transformation technology. The method presented here is also simple to use for obtaining numerical solution of the equations without using any discrete techniques, and leads to high accuracy as well.

2. Basic idea of He's homotopy perturbation method

To illustrate the basic ideas of this method, we consider the following nonlinear differential equation [8]:

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{2.1}$$

with the boundary conditions of

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma,$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω.

Generally speaking, the operator A can be decomposed into two operators, L and N, where L is linear, and N is nonlinear operator. Equation (2.1) can therefore be rewritten as follows:

$$L(u) + N(u) - f(r) = 0.$$

By the homotopy technique, we construct a homotopy $V\colon \Omega \times [0,1] \to \mathbb{R}$ and let:

$$H(V,p) = (1-p)[L(u) - l(u_0)] + p[A(V) - f(r)] = 0, \quad p \in [0,1], \ r \in \Omega \tag{2.2}$$

or

$$H(V,p) = L(u) - l(u_0) + pl(u_0) + p[N(V) - f(r)] = 0, \tag{2.3}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is an initial approximation of equation (2.1), which satisfies the boundary conditions. Obviously, from (2.2) and (2.3), we will have:

$$H(V,0) = L(u) - l(u_0) = 0, \quad H(V,1) = A(V) - f(r) = 0.$$

The changing process of p from zero to unity is just that of $V(r,p)$ from $u_0(r)$ to $u(r)$.

According to the HPM, we can first use the embedding parameter p as a "small parameter", and assume that the solution of equations (2.2) and (2.3) can be written as a power series in p:

$$V = V_0 + pV_1 + p^2V_2 + \dots$$

Setting $p = 1$ results in the approximate solution of (2.1):

$$V = \lim_{p=1} V = V_0 + V_1 + V_2 + \dots \tag{2.4}$$

The combination of the perturbation method and the homotopy method is called the homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantage of the traditional perturbation techniques.

The series (2.4) is convergent for most cases.

3. Analysis of the method

To investigate the traveling wave solution of equation (1.1), we first construct a homotopy as follows:

$$(1-p)\left(\frac{\partial u}{\partial t} - \frac{\partial u_0}{\partial t}\right) + p\left(\frac{\partial u}{\partial t} - \frac{\partial^3 u}{2\partial x^3} + 3u^2\frac{\partial u}{\partial x} - \frac{3\partial^2 v}{\partial x^2} - \frac{3\partial uv}{\partial x} + 3\lambda\frac{\partial u}{\partial x}\right) = 0, \tag{3.1}$$

$$(1-p)\left(\frac{\partial v}{\partial t} - \frac{\partial v_0}{\partial t}\right) + p\left(\frac{\partial v}{\partial t} + \frac{\partial^3 v}{\partial x^3} + 3v\frac{\partial v}{\partial x} + 3\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} - 3u^2\frac{\partial v}{\partial x} - 3\lambda\frac{\partial v}{\partial x}\right) = 0. \tag{3.2}$$

Suppose the solution of equations (3.1)–(3.2) and the initial approximations are as follows:

$$u_0(x,t) = u(x,0), \quad v_0(x,t) = v(x,0), \tag{3.3}$$

$$u(x,t) = U(x,t) = u_0 + pu_1 + p^2u_2 + p^3u_3 + \dots, \tag{3.4}$$

$$v(x,t) = V(x,t) = v_0 + pv_1 + p^2v_2 + p^3v_3 + \dots. \tag{3.5}$$

where u_i, v_i, $(i = 1, 2, \dots)$ are functions of (x,t), yet to be determined. Substituting (3.4)–(3.5) into (3.1), and equating the coefficients of the terms with the identical powers of p, we have

$$\begin{aligned}
&\left(\frac{\partial u_0}{\partial t}+3u_0^2\frac{\partial u_0}{\partial x}-3u_0\frac{\partial v_0}{\partial x}+3\lambda\frac{\partial u_0}{\partial x}-\frac{3\partial^2 v_0}{\partial x^2}+\frac{\partial u_1}{\partial t}-3v_0\frac{\partial u_0}{\partial x}-\frac{\partial^3 u_0}{2\partial x^3}\right)p\\
&\quad+\left(\frac{\partial u_2}{\partial t}-\frac{\partial^3 u_1}{2\partial x^3}-3u_1\frac{\partial v_0}{\partial x}+3\lambda\frac{\partial u_1}{\partial t}-3v_0\frac{\partial u_1}{\partial x}\right.\\
&\quad\left.+3u_0^2\frac{\partial u_1}{\partial x}-\frac{3\partial^2 v_1}{\partial x^2}-3u_0\frac{\partial v_1}{\partial x}+6u_0u_1\frac{\partial u_0}{\partial x}-3v_1\frac{\partial u_0}{\partial x}\right)p^2\\
&\quad+\left(\frac{\partial u_3}{\partial t}-3v_1\frac{\partial u_1}{\partial x}-\frac{3\partial^2 u_2}{2\partial x^2}+3u_1^2\frac{\partial u_0}{\partial x}-3v_0\frac{\partial u_2}{\partial x}\right.\\
&\quad-\frac{\partial^3 u_2}{\partial x^3}-3u_2\frac{\partial v_0}{\partial x}-3v_2\frac{\partial u_0}{\partial x}+6u_0u_1\frac{\partial u_1}{\partial x}+3u_0^2\frac{\partial u_2}{\partial x}\\
&\quad\left.-3u_0\frac{\partial v_2}{\partial x}+3\lambda\frac{\partial u_2}{\partial x}-3u_1\frac{\partial v_1}{\partial x}+6u_0u_2\frac{\partial u_0}{\partial x}\right)p^3+\ldots=0,\\
&\left(\frac{\partial v_1}{\partial t}+\frac{\partial v_0}{\partial t}+3v_0\frac{\partial v_0}{\partial x}+\frac{\partial^3 v_0}{\partial x^3}-3\lambda\frac{\partial v_0}{\partial x}+3\frac{\partial u_0}{\partial x}\frac{\partial v_0}{\partial x}-3u_0^2\frac{\partial v_0}{\partial x}\right)p\\
&\quad+\left(\frac{\partial v_2}{\partial t}+3v_0\frac{\partial v_1}{\partial x}+3\frac{\partial u_1}{\partial x}\frac{\partial v_0}{\partial x}+\frac{\partial^3 v_1}{\partial x^3}-3\lambda\frac{\partial v_1}{\partial x}\right.\\
&\quad\left.-3u_0^2\frac{\partial v_1}{\partial x}+3v_1\frac{\partial v_0}{\partial x}-6u_0u_1\frac{\partial v_0}{\partial x}+3\frac{\partial u_0}{\partial x}\frac{\partial v_1}{\partial x}\right)p^2\\
&\quad+\left(\frac{\partial v_3}{\partial t}-3\lambda\frac{\partial v_2}{\partial x}+3\frac{\partial v_0}{\partial x}\frac{\partial u_2}{\partial x}+3v_2\frac{\partial v_0}{\partial x}-3u_0^2\frac{\partial v_2}{\partial x}\right.\\
&\quad+3v_1\frac{\partial v_1}{\partial x}-3u_1^2\frac{\partial v_0}{\partial x}+3\frac{\partial v_2}{\partial x}\frac{\partial u_0}{\partial x}+3v_0\frac{\partial v_2}{\partial x}\\
&\quad\left.-6u_0u_1\frac{\partial v_1}{\partial x}+\frac{\partial^3 v_2}{\partial x^3}-6u_0u_2\frac{\partial v_0}{\partial x}+3\frac{\partial v_1}{\partial x}\frac{\partial u_1}{\partial x}\right)p^3+\ldots=0.
\end{aligned}$$

In order to obtain the unknowns of u_i, v_i, $(i=1,2,\ldots)$, we have to construct and solve the following system which includes six equations with six unknowns, considering the initial approximations of equations (3.1)–(3.2)

$$\begin{aligned}\frac{\partial u_0}{\partial t}+3u_0^2\frac{\partial u_0}{\partial x}-3u_0\frac{\partial v_0}{\partial x}+3\lambda\frac{\partial u_0}{\partial x}&\\
-\frac{3\partial^2 v_0}{\partial x^2}+\frac{\partial u_1}{\partial t}-3v_0\frac{\partial u_0}{\partial x}-\frac{\partial^3 u_0}{2\partial x^3}&=0,\end{aligned}\tag{3.6}$$

$$\begin{aligned}\frac{\partial u_2}{\partial t}-\frac{\partial^3 u_1}{2\partial x^3}-3u_1\frac{\partial v_0}{\partial x}+3\lambda\frac{\partial u_1}{\partial t}-3v_0\frac{\partial u_1}{\partial x}&\\
+3u_0^2\frac{\partial u_1}{\partial x}-\frac{3\partial^2 v_1}{\partial x^2}-3u_0\frac{\partial v_1}{\partial x}+6u_0u_1\frac{\partial u_0}{\partial x}-3v_1\frac{\partial u_0}{\partial x}&=0,\end{aligned}\tag{3.7}$$

$$
\begin{aligned}
(3.8)\quad & \frac{\partial u_3}{\partial t} - 3v_1\frac{\partial u_1}{\partial x} - \frac{3\partial^2 u_2}{2\partial x^2} + 3u_1^2\frac{\partial u_0}{\partial x} - 3v_0\frac{\partial u_2}{\partial x} \\
& - \frac{\partial^3 u_2}{\partial x^3} - 3u_2\frac{\partial v_0}{\partial x} - 3v_2\frac{\partial u_0}{\partial x} + 6u_0u_1\frac{\partial u_1}{\partial x} + 3u_0^2\frac{\partial u_2}{\partial x} \\
& - 3u_0\frac{\partial v_2}{\partial x} + 3\lambda\frac{\partial u_2}{\partial x} - 3u_1\frac{\partial v_1}{\partial x} + 6u_0u_2\frac{\partial u_0}{\partial x} = 0,
\end{aligned}
$$

$$
(3.9)\quad \frac{\partial v_1}{\partial t} + \frac{\partial v_0}{\partial t} + 3v_0\frac{\partial v_0}{\partial x} + \frac{\partial^3 v_0}{\partial x^3} - 3\lambda\frac{\partial v_0}{\partial x} + 3\frac{\partial u_0}{\partial x}\frac{\partial v_0}{\partial x} - 3u_0^2\frac{\partial v_0}{\partial x} = 0,
$$

$$
\begin{aligned}
(3.10)\quad & \frac{\partial v_2}{\partial t} + 3v_0\frac{\partial v_1}{\partial x} + 3\frac{\partial u_1}{\partial x}\frac{\partial v_0}{\partial x} + \frac{\partial^3 v_1}{\partial x^3} \\
& - 3\lambda\frac{\partial v_1}{\partial x} - 3u_0^2\frac{\partial v_1}{\partial x} + 3v_1\frac{\partial v_0}{\partial x} - 6u_0u_1\frac{\partial v_0}{\partial x} + 3\frac{\partial u_0}{\partial x}\frac{\partial v_1}{\partial x} = 0,
\end{aligned}
$$

$$
\begin{aligned}
(3.11)\quad & \frac{\partial v_3}{\partial t} - 3\lambda\frac{\partial v_2}{\partial x} + 3\frac{\partial v_0}{\partial x}\frac{\partial u_2}{\partial x} + 3v_2\frac{\partial v_0}{\partial x} - 3u_0^2\frac{\partial v_2}{\partial x} + 3v_1\frac{\partial v_1}{\partial x} - 3u_1^2\frac{\partial v_0}{\partial x} \\
& + 3\frac{\partial v_2}{\partial x}\frac{\partial u_0}{\partial x} + 3v_0\frac{\partial v_2}{\partial x} - 6u_0u_1\frac{\partial v_1}{\partial x} + \frac{\partial^3 v_2}{\partial x^3} - 6u_0u_2\frac{\partial v_0}{\partial x} + 3\frac{\partial v_1}{\partial x}\frac{\partial u_1}{\partial x} = 0.
\end{aligned}
$$

If the first three approximations are sufficient, we will obtain:
(3.12)

$$
u(x,t) = \lim_{p\to 1} U(x,t) = \sum_{k=0}^{k=3} u_k(x,t), \qquad v(x,t) = \lim_{p\to 1} V(x,t) = \sum_{k=0}^{k=3} v_k(x,t).
$$

4. Application

Firstly, we consider the solutions of (1.1) with the initial conditions (see [3]):

$$
(4.1)\qquad u_0 = \frac{b}{2k} + k\tanh(kx), \qquad v_0 = \frac{\lambda}{2}\left(1 + \frac{k}{b}\right) + b\tanh(kx),
$$

where k and b are arbitrary constants. To calculate the terms of the homotopy series (3.12) for $u(x,t)$ and $v(x,t)$, we substitute the initial conditions (4.1) and (3.12) into the system (3.6)–(3.11), the solutions can be obtained as follows:

$$
\begin{aligned}
(4.2)\qquad & u_0(x,t) = u_0(x,0) = u(x,0) = \frac{b}{2k} + k\tanh(kx), \\
(4.3)\qquad & u_1(x,t) = \frac{(6\lambda k^2 b + 4k^4 b - 3b^3 - 6k^3\lambda)[\tanh^2(kx) - 1]t}{4b}, \\
(4.4)\qquad & u_2(x,t) = \{12[3k\lambda - 2b(3\lambda + 2k^2)] + 9b^3(1 - 4\lambda k^3) \\
& \qquad + 4k^4b^2[3b^2(3\lambda - 2k^2) + (9\lambda^2 + 12\lambda k^2 + 4k^4)]\} \\
& \qquad \cdot \frac{[\tanh^2(kx) - 1]\tanh(kx)t^2}{16kb^2},
\end{aligned}
$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$v_0(x,t) = v_0(x,0) = v(x,0) = \frac{\lambda}{2}\left(1+\frac{k}{b}\right) + b\tanh(kx), \tag{4.5}$$

$$v_1(x,t) = \frac{(4k^4b - 6k^2\lambda b - 3b^3 + 6k^3\lambda)(\tanh^2(kx)-1)t}{4k}, \tag{4.6}$$

$$\begin{aligned} v_2(x,t) = \{&288bk^6\lambda(k-b)\tanh^5(kx) + 72b^2k^4\lambda(b-k)\tanh^4(kx) \\ &+ [9b^6 + 12k^2b^4(3\lambda - 2k^2) - 36\lambda k^3b^3 \\ &+ 4k^4b^2(9\lambda^2 + 4k^4 + 132\lambda k^2) \\ &- 24\lambda k^5 b(3\lambda + 22k^2) + 36k^6\lambda^2]\tanh^3(kx) \\ &+ 144b^2k^4\lambda(b-k)\tanh^2(kx) \\ &+ [9b^6 + 12k^2b^4(3\lambda - 2k^2) - 36k^3\lambda b^3 \\ &+ 4k^4b^2(9\lambda + 4k^4 + 60k^2\lambda) - 24\lambda k^5 b(3\lambda + 10k^2) \\ &+ 36k^6\lambda^2]\tanh(kx) + 72b^2k^4\lambda(k-b)\}\frac{t^2}{16bk^2}, \end{aligned} \tag{4.7}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

In this way the other components can be easily obtained. Substituting equations (4.2)–(4.7) into (3.12):

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots, \\ v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t) + v_3(x,t) + \cdots. \end{aligned}$$

we can obtain the closed form solutions as follows:

$$\begin{aligned} u(x,t) &= \frac{b}{2k} + k\tanh\left[k\left(x + \frac{1}{4}\left(\frac{6k\lambda}{b} + \frac{3b^2}{k^2} - 6\lambda - 4k^2\right)t\right)\right], \\ v(x,t) &= \frac{\lambda}{2}\left(1+\frac{k}{b}\right) + b\tanh\left[k\left(x + \frac{1}{4}\left(\frac{6k\lambda}{b} + \frac{3b^2}{k^2} - 6\lambda - 4k^2\right)t\right)\right]. \end{aligned}$$

The kink-type solitary wave solutions are in full agreement with the ones constructed by Fan [3].

To examine the accuracy and reliability of the HPM for the coupled MKdV system, we can also consider the different initial values [3]:

$$\begin{aligned} u(x,0) &= k\tanh(kx), \\ v(x,0) &= \frac{1}{2}(4k^2 + \lambda) - 2k^2\tanh^2(kx). \end{aligned} \tag{4.8}$$

where k is an arbitrary constant. To calculate the terms of the homotopy series (3.12) for $u(x,t)$ and $v(x,t)$, we substitute the initial conditions (4.8) and equations (3.12) into system (3.6)–(3.11), the solutions of the system can be obtained. Following this procedure as illustrated in the first example, we obtain the closed form of soliton solutions:

$$u(x,t) = \tanh\Big[k(x - \frac{1}{2}(3\lambda + 2k^2)t)\Big],$$
$$v(x,t) = \frac{1}{2}(\lambda + 4k^2) - 2k^2\tanh^2\Big[k\Big(x - \frac{1}{2}(3\lambda + 2k^2)t\Big)\Big].$$

In this case, solitary wave solutions of (1.1) of kink-type for $u(x,t)$, but bell-type for $v(x,t)$ were justified by Fan [3].

5. Comparing the results with the exact solutions

To demonstrate the convergence of the HPM, the results of the numerical examples are presented and only few terms are required to obtain accurate solutions. The accuracy of the HPM for the coupled MKdV system is controllable, and absolute errors are very small. These results are listed in Tables 1 and 2, it is seen that the implemented method achieves a minimum accuracy for the first three approximations for initial conditions (4.1). For the initial conditions (4.8), we need mor terms for $v(x,t)$ to improve its accuracy. It is also evident that when more terms are computed the numerical results get much closer to the corresponding exact solutions.

(x,t)	$\lvert u_{\text{exact}} - u_{\text{homotopy}}\rvert$	$\lvert v_{\text{exact}} - v_{\text{homotopy}}\rvert$
(0.1,0.1)	2.413610343E-8	2.413610343E-8
(0.1,0.2)	3.663037028E-7	3.663037028E-7
(0.1,0.3)	1.754575181E-6	1.754575181E-6
(0.2,0.1)	4.574557069E-8	4.574557069E-8
(0.2,0.2)	7.144966135E-7	7.144966135E-7
(0.2,0.3)	3.538101206E-6	3.538101206E-6
(0.3,0.1)	6.015304680E-8	6.015304680E-8
(0.3,0.2)	9.523327446E-7	9.523327446E-7
(0.3,0.3)	4.773953444E-6	4.773953444E-6

TABLE 1. The HPM results for $u(x,t), v(x,t)$ for the first three approximations in comparison with the analytical solutions when $b = 1$, $k = 1$, $\lambda = 1$, for the solitary wave solutions with the initial conditions (4.1) of equations (1.1), respectively

6. Conclusions

In this paper, the homotopy perturbation method (HPM) was used for finding soliton solutions of a coupled MKdV system with initial conditions. It can be

(x,t)	$\lvert u_{\text{exact}} - u_{\text{homotopy}}\rvert$	$\lvert v_{\text{exact}} - v_{\text{homotopy}}\rvert$
(1,0.01)	7.583561082E-8	0.3806588329
(1,0.02)	1.291668628E-6	0.3979604164
(1,0.03)	6.953972064E-6	0.4148836580
(2,0.01)	4.624794765E-8	0.07479221354
(2,0.02)	7.484818668E-7	0.08062151984
(2,0.03)	3.829091662E-6	0.08686390626
(3,0.01)	8.285192424E-9	0.01065463095
(3,0.02)	1.341588331E-7	0.01153381026
(3,0.03)	6.899929688E-7	0.01248327260

TABLE 2. The HPM results for $u(x,t), v(x,t)$ for the first three approximation in comparison with the analytical solutions when $k = 1, \lambda = 1$, for the solitary wave solutions with the initial conditions (4.8) of equations (1.1), respectively

concluded that the HPM is a very powerful and efficient technique in finding exact solutions for wide classes of problems. It is worth pointing out that the HPM presents a rapid convergence for the solutions. The obtained solutions are compared with the Adomian decomposition method [11], and all examples show that the results of the present method are in excellent agreement with those obtained by the Adomian decomposition method. The HPM has many merits and much advantages over the Adomian decomposition method. This method is to overcome the difficulties arising in calculation of the Adomian polynomials. Also the HPM does not require small parameters in equations, so that the limitations of the traditional perturbation methods can be eliminated, and also the calculation in the HPM is simple and straightforward. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. The results show that the HPM is a powerful mathematical tool for solving systems of nonlinear partial differential equations.

REFERENCES

[1] J. BIAZAR , M. ESLAMI AND H. GHAZVINI, *Homotopy perturbation method for systems of partial differential equations*, Internat. J. Nonlinear Sci. Num. Simul. **8** (2007), 413–418.

[2] D. B. CAO, J. R. YAN AND Y. ZHANG, *Exact solutions for a new coupled MKdV equations and a coupled KdV equations*, Phys. Lett. A (2002), 68–71.

[3] E. FAN, *Soliton solutions for a generalized Hirota–Satsuma coupled KdV equation and a coupled MKdV equation*, Phys. Lett. A **282** (2001), 18–22.

[4] A. Ghorbani and J. Saberi-Nadjafi, *He's homotopy perturbation method for calculating Adomian polynomials*, Internat. J. Nonlinear Sci. Num. Simul. **8** (2007), 229–232.

[5] M. Gorji, D. D. Ganji and S. Soleimani, *New application of He's homotopy perturbation method*, Internat. J. Nonlinear Sci. and Num. Simul. **8** (2007), 319–328.

[6] M. M. Hassana and A. H. Khaterb, *Jacobi elliptic function solutions of three coupled nonlinear physical equations*, Z. Naturforsch **60a** (2005), 237–244.

[7] J. H. He, *Homotopy perturbation method for bifurcation of nonlinear problems*, Internat. J. Nonlinear Sci. Num. Simul. **6** (2005), 207–208.

[8] ———, *Application of homotopy perturbation method to nonlinear wave equations*, Chaos Solitons Fractals (2005), 695–700.

[9] W. Malfliet, *Solitary wave solutions of nonlinear wave equations*, Amer. J. Phys. **60** (1992), 148–160.

[10] E. J. Parks and B. R. Duffy, *An automated* tanh-*function method for finding solitary wave solutions to non-linear evolution equations*, Math. Appl. Mech. Enrg. **98** (1996), 288–300.

[11] K. R. Raslan, *The decomposition method for a Hirota–Satsuma coupled KdV equation and a coupled MKdV equation*, Internat. J. Comput. Math. **81** (2004), 1497–1505.

[12] Y. T. Wu, X. G. Geng, X. B. Hu and S. M. Zhu, *A generalized Hirota–Satsuma coupled Korteweg–de Vries equation and Miurs transformation*, Phys. Lett. A **255** (1999), 259–264.

Manuscript received September 18, 2007

Yong-Qing Jiang and Jia-Min Zhu
Department of Physics
Zhejiang Lishui University
Lishui 323000, P. R. CHINA

E-mail address: lsjyq28@163.com, zjm64@163.com