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MULTIPLICITY RESULTS FOR SUPERQUADRATIC DIRICHLET BOUNDARY VALUE PROBLEMS IN $\mathbb{R}^2$

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Abstract. In this paper it is studied the Dirichlet problem associated to the planar system $z' = J\nabla F(t,z)$. We consider the situation where the Hamiltonian F satisfies a superquadratic-type condition at infinity.

By means of a bifurcation argument we prove the existence of infinitely many solutions. These solutions are distinguished by the Maslov index of an associated linear system.

1. Introduction

We are concerned with a boundary value problem of the form

$$\begin{cases} z' = J\nabla F(t,z), & z = (x,y) \in \mathbb{R}^2,\ t \in [0,\pi], \\ x(0) = 0 = x(\pi), \end{cases} \tag{1.1}$$

where J is the standard symplectic matrix and $F \in C^2([0,\pi] \times \mathbb{R}^2; \mathbb{R}^+)$ satisfies $\nabla F(t,0) = 0$, for every $t \in [0,\pi]$. We prove a multiplicity result in the case when

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the nonlinear term F in (1.1) has a superquadratic growth. More precisely, we assume in what follows that

$$\lim_{|z|\to\infty} \underline{\lambda}_H(t,z) = \infty, \quad \text{uniformly in } t \in [0,\pi],$$

where $\underline{\lambda}_H(t,z)$ is the minimum eigenvalue of $H(t,z)$, the Hessian matrix of $F(t,z)$.

The problem of the existence of multiple solutions to superquadratic problems has been widely studied in the last decades.

In the framework of variational methods, Long [9] (generalizing a previous result of Bahri–Berestycki [3]) has proved the existence of infinitely many *periodic* solutions to an hamiltonian system in $\mathbb{R}^{2N}$, $N \geq 1$, of the form $\nabla F(t,z) = \nabla H(z) + f(t)$, being H a C^1-function satisfying the Rabinowitz superlinearity condition and such that there exist $1 < p_1 \leq p_2 < 2p_1 + 1$, $\alpha_i > 0$, $\beta_i \geq 0$, $i = 1,2$ for which

$$\alpha_1 |z|^{p_1+1} - \beta_1 \leq H(z) \leq \alpha_2 |z|^{p_2+1} + \beta_2, \quad \text{for all } z \in \mathbb{R}^{2N}.$$

In the particular case when system (1.1) arises from a second order scalar equation, several results are available in the literature. Here, we restrict ourselves to considering some contributions based on an abstract bifurcation argument; we refer indeed to the seminal paper by Rabinowitz [15] and, among others, to the results by Esteban [8] and Rynne [16] (in which the case of $2m$-th order equations is also considered).

In these papers, multiplicity is achieved by means of estimates on the number of zeros in $[0,\pi]$ of (possible) solutions to second order equations.

In more recent years, some of the techniques developed in the above cited works have been imported to the study of higher order equations, systems in $\mathbb{R}^2$ and (under serious restrictions) in $\mathbb{R}^{2N}$, $N \geq 1$, as well. More precisely, Ward [17], [18] has used the concept of rotation number (cf. (2.9)) for the obtention of a multiplicity result for planar systems. See also the work of C. Bereanu [4].

In [17], the proof is based on a global bifurcation result for a system of the form

$$\begin{cases} z' = \lambda J z + g(\lambda, t, z), \\ x(0) = 0 = x(\pi), \end{cases} \tag{1.2}$$

being $g(\lambda,t,z) = o(|z|)$, $|z| \to 0$.

Our result (Theorem 2.2) is based on a global bifurcation theorem that we obtained in [5]. In that paper, we proved the existence of global continua of solutions to systems in $\mathbb{R}^{2N}$, $N \geq 1$, of the form

$$\begin{cases} z' = \lambda J S_0(t) z + J G(t,z) z, \\ x(0) = 0 = x(\pi), \end{cases} \tag{1.3}$$

along which the Maslov index (cf. [1], [2]) is preserved.

In what follows, we assume that the Hessian matrix of F is positive definite (cf. Remark 2.9). In this situation, the Maslov index is well-defined; moreover, in the particular case of planar systems it coincides with the rotation number. We are thus able to perform concrete estimates which lead (as an application of the abstract bifurcation theorem) to the existence of infinitely many solutions to (1.1).

2. Main result

Let us consider the Dirichlet problem

$$\begin{cases} z' = J \nabla F(t,z), \qquad z = (x,y) \in \mathbb{R}^2, \\ x(0) = 0 = x(\pi), \end{cases} \tag{2.1}$$

where $F \in C^2([0,\pi] \times \mathbb{R}^2; \mathbb{R}^+)$ satisfies $\nabla F(t,0) = 0$, for every $t \in [0,\pi]$. We denote by $H(t,z)$ the Hessian matrix of $F(t,z)$ and by $\underline{\lambda}_H(t,z)$ the minimum eigenvalue of $H(t,z)$; we assume that $H(t,0)$ is of class C^1, $H(t,z)$ is positive definite and

$$\lim_{|z| \to \infty} \underline{\lambda}_H(t,z) = \infty, \quad \text{uniformly in } t \in [0,\pi]. \tag{2.2}$$

We observe that (2.2) is a superquadratic assumption on the Hamiltonian F. Moreover, we suppose that there exist $C > 0$ and $Z > 0$ such that

$$\left| \frac{\partial F}{\partial t}(t,z) \right| \leq C F(t,z), \quad \text{for all } t \in [0,\pi],\ |z| \geq Z. \tag{2.3}$$

EXAMPLE 2.1. We give an explicit example of a function F satisfying the above assumptions.

Let us consider a function $\alpha \in C^2([0,\pi])$ such that

$$\alpha(t) > \alpha_0 > 0, \quad \text{for all } t \in [0,\pi]. \tag{2.4}$$

Moreover, for every $z = (z_1, z_2) \in \mathbb{R}^2$, let

$$A(z) = z_1 |z_1|^{p-1} + z_2 |z_2|^{q-1} + \beta(z),$$

where $p > 2$, $q > 2$ and $\beta \in C^2(\mathbb{R}^2; \mathbb{R}^+)$ is a scalar field whose Hessian is positive definite and such that $\nabla \beta(0) = 0$. Then, the function F defined by

$$F(t,z) = \alpha(t) A(z), \quad \text{for all } t \in [0,\pi],\ z \in \mathbb{R}^2,$$

satisfies the assumptions of Theorem 2.2.

Indeed, $F \in C^2([0,\pi] \times \mathbb{R}^2; \mathbb{R}^+)$ and $\nabla F(t,0) = \alpha(t)\nabla A(0) = 0$, for every $t \in [0,\pi]$. Moreover, the Hessian matrix of $F(t,z)$ is

$$H(t,z) = \alpha(t) H_A(z),$$

where H_A denotes the Hessian matrix of A. It is easy to check that

$$H_A(z) = H^*(z) + H_\beta(z),$$

where

$$H^*(z) = \begin{pmatrix} p(p-1)|z_1|^{p-2} & 0 \\ 0 & q(q-1)|z_2|^{q-2} \end{pmatrix}$$

and H_β is the Hessian matrix of β. From the fact that the Hessian matrix of β is positive definite we deduce that

$$H_A(z) > H^*(z), \quad \text{for all } z \in \mathbb{R}^2,$$

in the sense that the difference between the two matrices is positive definite. As a consequence, from (2.4) we infer that

$$H(t,z) > \alpha_0 H^*(z), \quad \text{for all } t \in [0,\pi],\ z \in \mathbb{R}^2.$$

This is sufficient to ensure that H is definite positive and that (2.2) holds true.

Finally, a straighforward computation, together with (2.4), proves that also (2.3) is fulfilled.

We observe that the boundary value problem (2.1) can be written in the form

$$\begin{cases} z' = JS(t,z)z, \\ x(0) = 0 = x(\pi), \end{cases} \tag{2.5}$$

where $S(t,z)$ is the symmetric matrix defined by

$$S(t,z) = \int_0^1 H(t,sz)\,ds, \quad \text{for all } t \in [0,\pi],\ z \in \mathbb{R}^2.$$

Moreover, it is easy to see that assumption (2.2) implies that

$$\lim_{|z|\to\infty} \underline{\lambda}_S(t,z) = \infty, \quad \text{uniformly in } t \in [0,\pi], \tag{2.6}$$

where $\underline{\lambda}_S(t,z)$ denotes the minimum eigenvalue of $S(t,z)$.

For every solution z of (2.5) let us consider the Maslov index ([1], [2], [5] and [13]) of the linear system

$$w' = JS(t,z(t))w, \quad w = (u,v),$$

with respect to the boundary conditions $u(0) = 0 = u(\pi)$. Under the above conditions, we will prove the following:

THEOREM 2.2. *There exists* $k_0 \in \mathbb{N}$ *such that for every integer* $k \geq k_0$ *there exists a solution* z_k *of* (2.1) *such that the Maslov index of* $w' = JS(t, z_k(t))w$ *is* $k - 1$.

REMARK 2.3. As for Hamiltonian systems, Theorem 2.2 improves Theorem 8 in [17]; indeed, in [17]–[19] the author studied a problem of the form (2.5) with S a diagonal matrix.

Both these results are obtained through a bifurcation argument based on the study of (1.3) and (1.2), respectively. However, it has to be pointed out that in [17] the periodic BVP and general separated boundary conditions are considered; moreover, in [17] an extension of the main result to some particular system in $\mathbb{R}^{2N}$ is treated as well. The reason why we mainly treat Hamiltonian systems is explained in Remark 2.6.

We also point out that the paper [9] deals with the *periodic* problem in $\mathbb{R}^{2N}, N \geq 1$; on the other hand, the nonlinearity we consider in (1.1) need not be a bounded perturbation of an autonomous function.

Finally, we observe that Theorem 2.2 is not comparable with results dealing with a second order scalar equation of the form $u'' + g(u) = p(t)$, with $\lim_{|x| \to \infty} g(x)/x = \infty$. In fact, under this condition the hypothesis (2.2) might fail.

In order to prove the result we use a bifurcation argument. First of all, we observe that $S(t, 0) := S_0(t)$ is positive definite; let us then consider the boundary value problem

$$\begin{cases} z' = \lambda JS_0(t)z + JG(t, z)z, \\ x(0) = 0 = x(\pi), \end{cases} \tag{2.7}$$

where $G(t, z) = S(t, z) - S_0(t)$ and $\lambda \in [1, \infty)$. We are interested in finding solutions of (2.7) with $\lambda = 1$.

Since the matrix $S_0(t)$ is positive definite, for every $t \in [0, \pi]$, there exists an unbounded sequence μ_k, $k \in \mathbb{Z}$, of (simple) eigenvalues of $z' = \lambda JS_0(t)z$, $x(0) = 0 = x(\pi)$, such that $\mu_k \to \pm\infty$, as $k \to \pm\infty$; in particular, there exists $k_0 \in \mathbb{N}$ such that $\mu_k > 1$, for every $k \geq k_0$.

From [5, Theorem 3.9] we deduce that every point of the form $(\mu_k, 0)$ is a global bifurcation point for (2.7); indeed, let us denote by Σ the closure of the set of nontrivial solutions of (2.7). Then, for every $k \geq k_0$, Σ contains an unbounded continuum C_k such that $(\mu_k, 0) \in C_k$ and $m(\lambda, z) = k - 1$, for every $(\lambda, z) \in C_k$, where $m(\lambda, z)$ is the Maslov index of the linear system

$$w' = \lambda JS_0(t)w + JG(t, z(t))w. \tag{2.8}$$

In this framework, the fact that C_k is unbounded means that one of the following conditions holds true:

(a) there exists $(\lambda_n, z_n) \in C_k$ such that $\lambda_n \to \infty$;
(b) there exists $(\lambda_n, z_n) \in C_k$ such that $||z_n|| \to \infty$;
(c) there exists $(\lambda_n, z_n) \in C_k$ such that $\lambda_n \to 1^+$.

We will show that (a) and (b) cannot occur; this implies that the bifurcating branch C_k must intersect the line $\lambda = 1$, giving rise to a solution to (2.1).

To this aim, let us first observe that in the case of planar systems the Maslov index is strictly related to the rotation number; indeed, let (λ, z) be a solution of (2.7) and let $w = (u, v)$ be the solution of (2.8) such that $(u(0), v(0)) = (0, 1)$. The rotation number of (2.8) is defined as

$$\mathrm{rot}(\lambda, z) = \frac{1}{\pi} \int_0^\pi \frac{u(t)v'(t) - u'(t)v(t)}{u(t)^2 + v(t)^2}\, dt; \tag{2.9}$$

if we denote by $q_{\lambda,z}$ the quadratic form associated to the symmetric matrix $\lambda S_0(t) + G(t, z(t))$, then it is easy to see that

$$\mathrm{rot}(\lambda, z) = \frac{1}{\pi} \int_0^\pi \frac{q_{\lambda,z}(u(t), v(t))}{u(t)^2 + v(t)^2}\, dt. \tag{2.10}$$

By adapting the argument in [10] it is possible to check that

$$m(\lambda, z) = \mathrm{rot}(\lambda, z) - 1.$$

Using this fact we are able to prove the following result:

Proposition 2.4. *There exists $M_k > 0$ such that for every $(\lambda, z) \in C_k$ we have $\lambda \le M_k$.*

Proof. For every $(\lambda, z) \in C_k$ we have $m(\lambda, z) = k - 1$ and

$$\mathrm{rot}(\lambda, z) = k. \tag{2.11}$$

We denote by $q_{(\lambda-1)S_0}$ and q_{S_z} the quadratic forms associated to the matrices $(\lambda - 1)S_0(t)$ and $S(t, z(t))$, respectively.

From assumption (2.6) we deduce that there exists $M > 0$ such that

$$q_{S_z}(u, v) \ge -M(u^2 + v^2), \quad \text{for all } (u, v) \in \mathbb{R}^2;$$

therefore we obtain

$$q_{\lambda,z}(u, v) \ge q_{(\lambda-1)S_0}(u, v) - M(u^2 + v^2), \quad \text{for all } (u, v) \in \mathbb{R}^2.$$

Moreover, since $S_0(t)$ is positive definite, we conclude that there exists $\lambda_0 > 0$ such that

$$q_{\lambda,z}(u, v) \ge ((\lambda - 1)\lambda_0 - M)(u^2 + v^2), \quad \text{for all } (u, v) \in \mathbb{R}^2. \tag{2.12}$$

From (2.10) and (2.12) we infer that

$$\mathrm{rot}(\lambda, z) \geq (\lambda - 1)\lambda_0 - M;$$

using now (2.11) we get

$$\lambda \leq \frac{k + M}{\lambda_0} + 1.$$

This concludes the proof. □

In order to exclude also alternative (b), we need to prove more properties of the solutions to (2.7) and of the Maslov index.

PROPOSITION 2.5. *Then, for every* $R_1 > 0$ *there exists* $R_2 > 0$ *such that for every solution* (λ, z) *of* (2.7) *we have*

$$\textit{if} \quad |z(0)| \leq R_1 \quad \textit{then} \quad |z(t)| \leq R_2, \quad \textit{for all } t \in [0, \pi].$$

PROOF. The proof is based on [7, Lemma 3]; accordingly, it is sufficient to find a positive function $V_\lambda \in C^1([0, \pi] \times \mathbb{R}^2)$, $\lambda \geq 1$, for which there exist $K > 0$ and $Z > 0$ such that

$$\left| \frac{\partial V_\lambda}{\partial t}(t, z) + \langle \nabla V_\lambda(t, z), \lambda J S_0(t) z + J G(t, z) z \rangle \right| \leq K V_\lambda(t, z), \tag{2.13}$$

for all $t \in [0, \pi]$, $|z| \geq Z$, $\lambda \geq 1$. To this aim, let

$$V_\lambda(t, z) = F(t, z) + \frac{1}{2}(\lambda - 1)\langle S_0(t) z, z\rangle + 1, \quad \text{for all } (t, z) \in [0, \pi] \times \mathbb{R}^2,\ \lambda \geq 1.$$

We observe that since $S_0(t)$ is positive definite, for every $t \in [0, \pi]$, then

$$|\langle S_0'(t) z, z\rangle| \leq c_1 ||z||^2 \leq c_2 \langle S_0(t) z, z\rangle, \quad \text{for all} \quad z \in \mathbb{R}^2, \tag{2.14}$$

for some positive constants c_1 and c_2; (2.14) and (2.3) ensure that there exists $c_3 > 0$ such that

$$\left| \frac{\partial V_\lambda}{\partial t}(t, z) \right| \leq c_3 V_\lambda(t, z), \quad \text{for all } t \in [0, \pi],\ |z| \geq Z,\ \lambda \geq 1. \tag{2.15}$$

Now, recalling that

$$\lambda J S_0(t) z + J G(t, z) z = (\lambda - 1) J S_0(t) z + J S(t, z) z = (\lambda - 1) J S_0(t) z + J \nabla F(t, z),$$

it is easy to see that

$$\begin{aligned} &\langle \nabla V_\lambda(t, z), \lambda J S_0(t) z + J G(t, z) z\rangle \\ &\qquad = \langle \nabla F(t, z) + (\lambda - 1) S_0(t) z, (\lambda - 1) J S_0(t) z + J \nabla F(t, z)\rangle = 0. \end{aligned} \tag{2.16}$$

From (2.15) and (2.16) we deduce that (2.13) holds true. □

REMARK 2.6. We observe that the fact that the system under consideration is Hamiltonian is used only in the proof of Proposition 2.5, in order to find a suitable function V_λ satisfying (2.13).

It is possible to prove a multiplicity result on the lines of Theorem 2.2 for a more general boundary value problem of the form (2.5); indeed, it is sufficient to require (2.3) and the existence of a "guiding-type" function V_λ (cf. [11]) for which (2.13) holds true.

Proposition 2.7. *For every* $N > 0$ *there exists* $Z_N > 0$ *such that for every solution* (λ, z) *of* (2.7) *we have*

$$\text{if} \quad |z(0)| \geq Z_N \quad \text{then} \quad m(\lambda, z) > N. \tag{2.17}$$

Proof. Let us denote by m_z and m_N the Maslov indices of the systems $w' = JS(t, z(t))w$ and $w' = (N+1)Jw$, respectively. It is immediate to see that $m_N = N$.

The monotonicity property of the Maslov index (see [14, Theorem 5.7]) ensures that

$$\text{if } S_0(t) \text{ is positive definite then } m(\lambda, z) \geq m_z. \tag{2.18}$$

Now, from (2.6) we deduce that there exists Z_N^* such that

$$\text{if} \quad |z| \geq Z_N^* \quad \text{then} \quad \underline{\lambda}_S(t, z) > N + 1, \quad \text{for all } t \in [0, \pi]. \tag{2.19}$$

Moreover, from Proposition 2.5 we infer that there exists Z_N such that for every solution (λ, z) of (2.7) we have

$$\text{if } |z(0)| \geq Z_N \quad \text{then} \quad |z(t)| \geq Z_N^*, \quad \text{for all } t \in [0, \pi]. \tag{2.20}$$

From (2.18)–(2.20) and using again the monotonicity property of the Maslov index we deduce that

$$\text{if} \quad |z(0)| \geq Z_N \quad \text{then} \quad m(\lambda, z) \geq m_z > m_N = N. \qquad \square$$

Using these results we can prove the following:

Proposition 2.8. *There exists* $R_k > 0$ *such that for every* $(\lambda, z) \in C_k$ *we have* $||u|| \leq R_k$.

Proof. By contradiction, assume that for every $R > R_2(Z_k)$ (with Z_k given in Proposition 2.7 and $R_2(Z_k)$ as in Proposition 2.5) there exists $(\lambda, z) \in C_k$ such that $||z|| \geq R$. For the solution z we necessarily have $|z(0)| \geq Z_k$; hence, from (2.17) we infer that $m(\lambda, z) > k$. This contradicts the fact that $m(\lambda, z) = k - 1$, for every $(\lambda, z) \in C_k$. $\square$

From Propositions 2.4 and 2.8 we are able to exclude conditions (a) and (b); as already observed, this proves Theorem 2.2.

Remark 2.9. Theorem 2.2 can be proved also in the case when $H(t, z)$ is not positive definite for every $(t, z) \in [0, \pi] \times \mathbb{R}^2$.

Indeed, the existence of a bifurcating unbounded branch C_k is obtained in [5] using the following facts:

(a) the fact that $H(t,z)$ is positive definite, for every $(t,z) \in [0,\pi] \times \mathbb{R}^2$, implies that it is possible to define the Maslov index of (2.8), for every solution (λ, z) of (2.7);

(b) the functional $\phi(\lambda,z) = m(\lambda,z)$ is defined and continuous on the set of all the solutions of (2.7).

When $H(t,z)$ is not positive definite for every $(t,z) \in [0,\pi] \times \mathbb{R}^2$, it is not possible, in general, to define $m(\lambda,z)$ for every solution (λ,z) of (2.7). However, in this situation we can define

$$\widetilde{\phi}(\lambda,z) = \begin{cases} m(\lambda,z) & \text{if defined,} \\ -1 & \text{otherwise.} \end{cases}$$

It is possible to see that $\widetilde{\phi}(\lambda,z) = -1$ if and only if $\operatorname{rot}(\lambda,z) = 0$; hence the relation

$$\widetilde{\phi}(\lambda,z) = \operatorname{rot}(\lambda,z) - 1, \quad \text{for all } (\lambda,z),$$

is satisfied and the continuity of $\widetilde{\phi}$ follows from the continuity of the rotation number.

REMARK 2.10. We observe that it is possible to obtain a multiplicity result on the lines of Theorem 2.2 by assuming instead of (2.2) the following

$$\lim_{||z||\to\infty} \overline{\lambda}_H(t,z) = -\infty, \quad \text{uniformly in } t \in [0,\pi],$$

where $\overline{\lambda}_H(t,z)$ is the maximum eigenvalue of $H(t,z)$.

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