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# OLGA ALEXANDROVNA LADYZHENSKAYA (1922–2004)

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In January 2004 the world mathematical community lost Professor Olga Alexandrovna Ladyzhenskaya who was a great mathematician and a member of several Academies of Science. She made significant and important contributions to the area of partial differential equations, particularly the Navier–Stokes equations, nonlinear elliptic and parabolic equations.

Olga Alexandrovna Ladyzhenskaya was born in 1922 in the tiny town Kologriv situated in the Kostromsk region of Russia. Her father, Alexander Ivanovich was a high school mathematics teacher and it was him who awoke Olga's interest in natural sciences. In 1937 he was arrested and shortly afterwards shot by NKVD as a "traitor". Due to Olga's father being prosecuted by the communist regime she was denied a place at Leningrad State University, though in 1939 she graduated with honors from high school and passed the entrance exams to this university. She was only accepted at Pokrovskii Pedagogical Institute in Leningrad whose she finished two years of studies. Forced by the war to relinquish her studies she became a high school teacher of mathematics amongst others in her home town. From 1943 she was a student of mathematics and mechanics at Moscow State University where she graduated in 1947. In the same year she married A. A. Kiselev and received a recommendation from Moscow University to the graduate school of Leningrad State University (LGU), where she took a post at the Mathematics and Mechanics Department. Her scientific adviser was S. L. Sobolev.

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Olga A. Ladyzhenskaya became a postgraduate student of V. I. Smirnov and it was on her request that he organized a seminar on mathematical physics and boundary-value problems. Later she became head of the seminar and was active there to the last days of her life.

In 1953, at Moscow State University, she defended her "habilitation" dissertation and in 1954 got an appointment in the Steklov Mathematical Institute in Leningrad. Since 1961 Olga A. Ladyzhenskaya as a leading member of the Department of Mathematical Physics engaged and cooperated with such mathematicians as: O. V. Guseva, V. A. Solonnikov, N. N. Uraltseva, L. D. Faddeyev, K. K. Golovkin, A. P. Oskolkov, A. V. Ivanov, Y. Ya. Rivkind, L. V. Kapitanskii, V. Shubov and many others.

Olga Alexandrovna Ladyzhenskaya passed away on January 12, 2004 in St. Petesburg (formerly Leningrad), after laborious life devoted to mathematics and other numerous activities.

There are two main directions in the scientific life of prof. Ladyzhenskaya. The first: the existence, uniqueness and regularity of solutions to the Navier–Stokes equations. The second: regularity theory for nonlinear elliptic and parabolic equations.

In 1951 Olga A. Ladyzhenskaya proved the second fundamental inequality for elliptic operators $L$ of second order with smooth coefficients and for any classical homogeneous boundary conditions (see [1]),

$$\|u\|_{W_2^2(\Omega)} \leq c_\Omega(\|Lu\|_{L_2(\Omega)} + \|u\|_{L_2(\Omega)}$$

which holds for any $u \in W_2^2(\Omega)$.

In the years 1951–1953 Olga A. Ladyzhenskaya wrote a series of papers devoted to convergence of the Fourier method for hyperbolic equations. The results were a subject of her monograph [2].

Concerning the first direction, in 1958 in [3] Olga A. Ladyzhenskaya proved the multiplicative inequality

$$\|u\|_{L_4(\Omega)}^4 \leq c\|u\|_{L_2(\Omega)}^2 \|\nabla u\|_{L_2(\Omega)}^2,$$

which holds for any $u \in \dot{W}_2^1(\Omega)$, $\Omega \in \mathbb{R}^2$.

This inequality gave a possibility to prove existence of global unique solution of two-dimensional Navier–Stokes system (see [3], [4]).

In three-dimensional case global existence for sufficiently small initial data and external force was proved with A. A. Kisielev [5].

In [6] Olga A. Ladyzhenskaya proved global existence of stationary solutions to Navier–Stokes equations in a bounded domain for the Dirichlet boundary conditions.

In [7] she showed global existence of regular axially symmetric solutions to Navier–Stokes system.

For a long time Olga A. Ladyzhenskaya examined the problem of regularity and uniqueness of the weak Hopf solutions. It was shown that if the weak solution $v$ belongs to $L_r(0,T;L_q(\Omega))$, $2/r+3/q\le 1$, $r\in[2,\infty)$, $q\in[3,\infty]$, or $r=\infty$, $q>3$, then the solution is unique and regular.

These results and other previous results were described in her monograph on Navier–Stokes [8].

Concerning the second direction, there were written two monographs [9] (with N. N. Uraltseva) and [10] (with V. A. Solonnikov and N. N. Uraltseva), in which the regularity problem for nonlinear elliptic and parabolic equations were examined. In these monographs profound local techniques (generalized methods of DeGiorgi, Morrey, Nash) were developed.

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