

Typically periodic optimization in ergodic optimization

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Ergodic optimization is the study of problems relating maximizing invariant measures and maximum ergodic averages. In ergodic optimization theory, one important problem is the typically periodic optimization (TPO) conjecture. This conjecture was proposed by Mañé[6], Hunt, Ott and Yuan[4, 7] in 1990s, which reveals the principle of least action in dynamical system settings. To be more precise, TPO indicates that when the dynamical system is suitably hyperbolic and the observable is suitably regular, then the maximizing measure is “genetically” supported on a periodic orbit with relatively low period. In the setting of uniformly open expanding maps with Lipschitz/Holder observables, TPO was obtained in topological genetic sense by Contreras [3] in 2016. In this series of talks, I will report several recent progresses on understanding TPO conjecture both in probabilistic and topological sense, and for more general hyperbolic systems. These are joint works with Jairo Bochi[1], Ding and Li [2], and Huang Wen, Zeng Lian, Xiao Ma, Leiye Xu [5].

References

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